Review

Sunday, February 25, 2007
3:19 PM

Dettman pg 62 #5

$\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$

$\sin z = i$

$\sin(x+iy) = \sin x \cos(iy) + \sin(iy)\cos x$

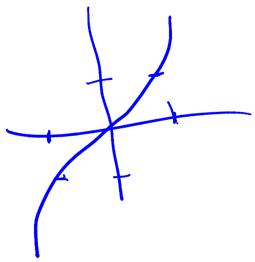
$= \sin x \cosh y + i \sinh y \cos x \overset{?}{=} i$

$\sin x \cosh y = 0 \Rightarrow \sin x = 0$ or ~~$\cosh y = 0$~~

$\sinh y \cos x = 1$

$\hookrightarrow x = n\pi$

$\cos n\pi = (-1)^n$



$\longrightarrow \cos n\pi \sinh y = 1$

$\sinh y = (-1)^n$

Case I n - even

$1 = \sinh y = \frac{e^y - e^{-y}}{2}$

$2 = e^y - e^{-y}$

$2e^y = (e^y)^2 - 1$

$0 = (e^y)^2 - 2e^y - 1$

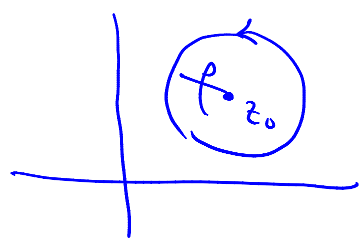
$e^y = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$

$y = \ln(1+\sqrt{2})$

$z = 2k\pi + i\ln(1+\sqrt{2}), \; k \in \mathbb{Z}$

pg 86 #6

$\int_C \frac{dz}{z-z_0}$, $C: |z-z_0| = \rho$

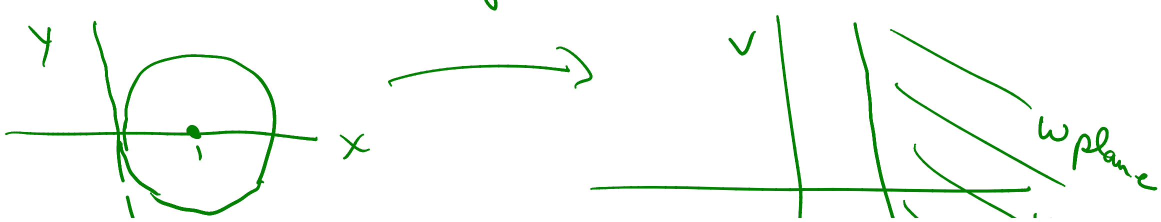


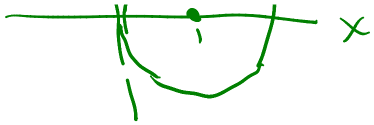
$z - z_0 = \rho e^{i\theta}$

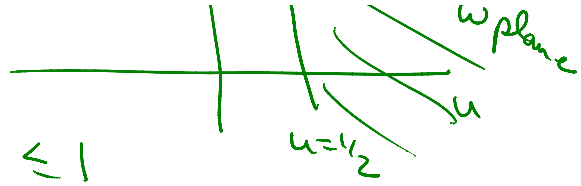
$dz = i\rho e^{i\theta} d\theta$

$\int_C \frac{dz}{z-z_0} = \int_0^{2\pi} \frac{i\rho e^{i\theta} d\theta}{\rho e^{i\theta}} = \int_0^{2\pi} i \, d\theta = 2\pi i$

pg 32 #1 Find the image of $|z-1| \le 1$ under $w = \frac{1}{z}$







$z = \frac{1}{w} \Rightarrow$

$$|z-1| \le 1$$
$$\left|\frac{1}{w} - 1\right| \le 1$$

Let $w = u + iv$

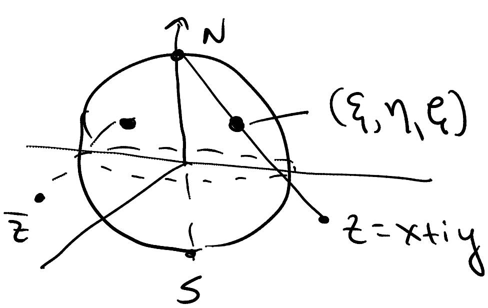
$$1 \ge \left|\frac{1}{w} - 1\right| = \left|\frac{1-w}{w}\right|$$
$$|w|^2 \ge |1-w|^2$$
$$u^2 + v^2 \ge |1-(u+iv)|^2$$
$$= (1-u)^2 + v^2$$
$$u^2 \ge 1 - 2u + u^2$$
$$0 \ge 1 - 2u \quad \text{or} \quad 2u \ge 1$$
$$u \ge 1/2$$

Pg 39 #7 $\quad w = \sqrt{z^2-1} = |z^2-1|^{1/2} e^{i(\arg(z^2-1)/2)}$ (at z_0)

$$w^2 = z^2 - 1$$
$$2ww' = 2z \quad @\ z = z_0 \qquad w_0 = w(z_0)$$
$$2w_0 w'(z_0) = 2z_0$$
$$w'(z_0) = \frac{z_0}{w_0} = \frac{z_0}{\sqrt{z_0^2-1}}$$

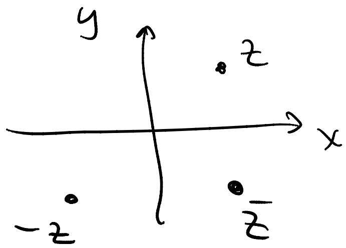
Pg 22 #1

$$\xi = \frac{2x}{r^2+1}, \quad \eta = \frac{2y}{r^2+1}, \quad \zeta = \frac{r^2-1}{r^2+1}$$



$\xi \to -\xi,\ \eta \to -\eta$ for $z \to -z$

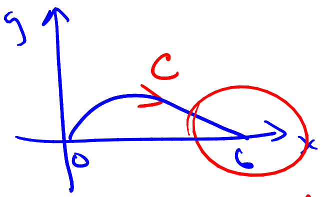
$$\frac{1}{z} = \frac{1}{z}\frac{\bar{z}}{\bar{z}} = \frac{1}{|z|^2}\bar{z}$$



−z (z

Pg 62 #129

Let $w = \sqrt{(4-z)(z^2+4)}$
if $w = 4$ when $z = 0$ show
if z describes C, the w at $z = 6$ is $-4i\sqrt{5}$

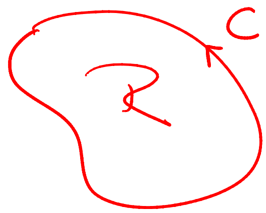


$$w = \sqrt{(4-6)(4+6^2)} = \sqrt{-80} = 4\sqrt{-5}$$
$$= \pm 4i\sqrt{5}$$

$z = |z| e^{i\theta}$

$z = 4 + a e^{i\theta}$

Pg 80 (Dettman)



$$A = \frac{1}{2}\int x\,dy - y\,dx$$

$$\int_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$

$$\frac{1}{2}\int x\,dy - y\,dx = \frac{1}{2}\iint_R [1-(-1)]\,dx\,dy$$
$$= \iint_R dx\,dy = A$$

Schaum's pg 58 #72

a) $\sinh 2z = u + iv$

$$\sinh 2z = \sinh 2(x+iy)$$
$$= \sinh 2x \cosh(2iy) + \sinh(2iy)\cosh 2x$$

$$\cosh 2iy = \frac{e^{2iy} + e^{-2iy}}{2} = \cos 2y$$

$$\sinh 2iy = \frac{e^{2iy} - e^{-2iy}}{2i}\, i = i \sin 2y$$

$$\sinh 2z = \sinh 2x \cos 2y + i \sin 2y \cosh 2x$$

http://www.alcyone.com/max/reference/maths/hyperbolic.html