How Does a PDE Chef Bake a Cake?

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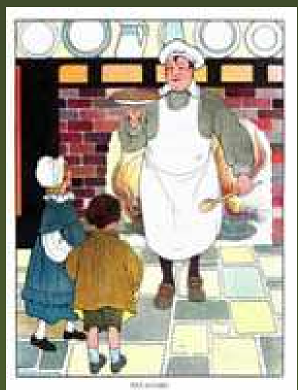
Abstract

While baking moist cakes and crusty breads is an art, one might think that modelling the process using the three dimensional heat equation should lead to good first order approximations. Is there a master recipe for a model of cake baking only to be improved by the touch of a numerical chef? We review some recent models in the literature and present preliminary models for the temperature distribution in cakes for different geometries.



http://www.apples4theteacher.com/mother-goose-nursery-rhymes/pat-a-cake.html

Pat-a-cake,
pat-a-cake,
Baker’s man!
So I do, master,
As fast as I can.

Pat it, and prick it,
And mark it with T,
Put it in the oven
For Tommy and me.

Outline

- Heat Equation
  - Rectangular Cake - Fixed BCs
  - Cylindrical Cake - Fixed BCs

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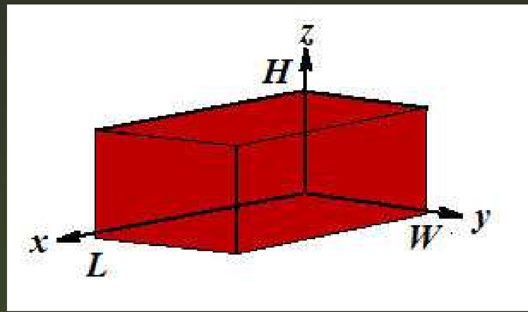
  - Moisture Content
    - Two Layer Model
    - Coupled PDEs

Rectangular Cakes



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$$\frac{\partial T}{\partial t} = D\nabla^2 T$$

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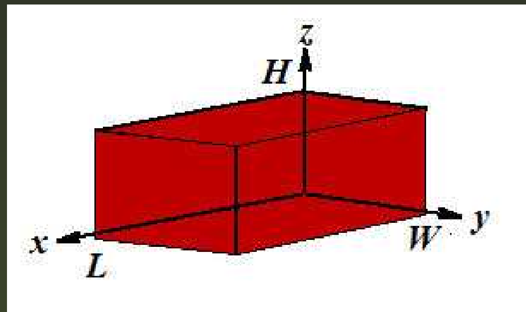

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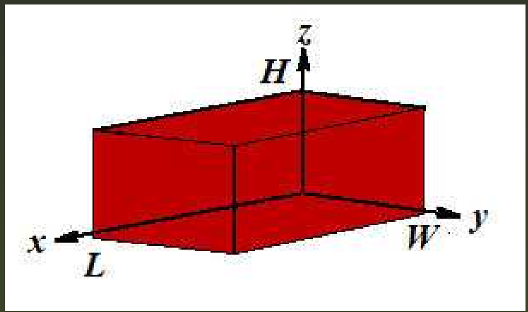

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Boundary Conditions: $T(\cdot) = T_b$ for

$$x = 0, L \qquad y = 0, W \qquad z = 0, H.$$

Solution - Rectangular Cake

$$T(t) = T_b + \frac{64(T_i - T_b)}{\pi^3} \sum_{n=1}^{\infty} \sum_{\ell=1}^{\infty} \sum_{m=1}^{\infty} \frac{\sin \frac{(2n-1)\pi x}{L} \sin \frac{(2\ell-1)\pi y}{W} \sin \frac{(2m-1)\pi z}{H}}{(2n-1)(2\ell-1)(2m-1)} e^{D\lambda_{n\ell m} t}$$

where $\lambda_{n\ell m} = -\left(\left(\frac{(2m-1)\pi}{L} \right)^2 + \left(\frac{(2\ell-1)\pi}{W} \right)^2 + \left(\frac{(2n-1)\pi}{H} \right)^2 \right).$

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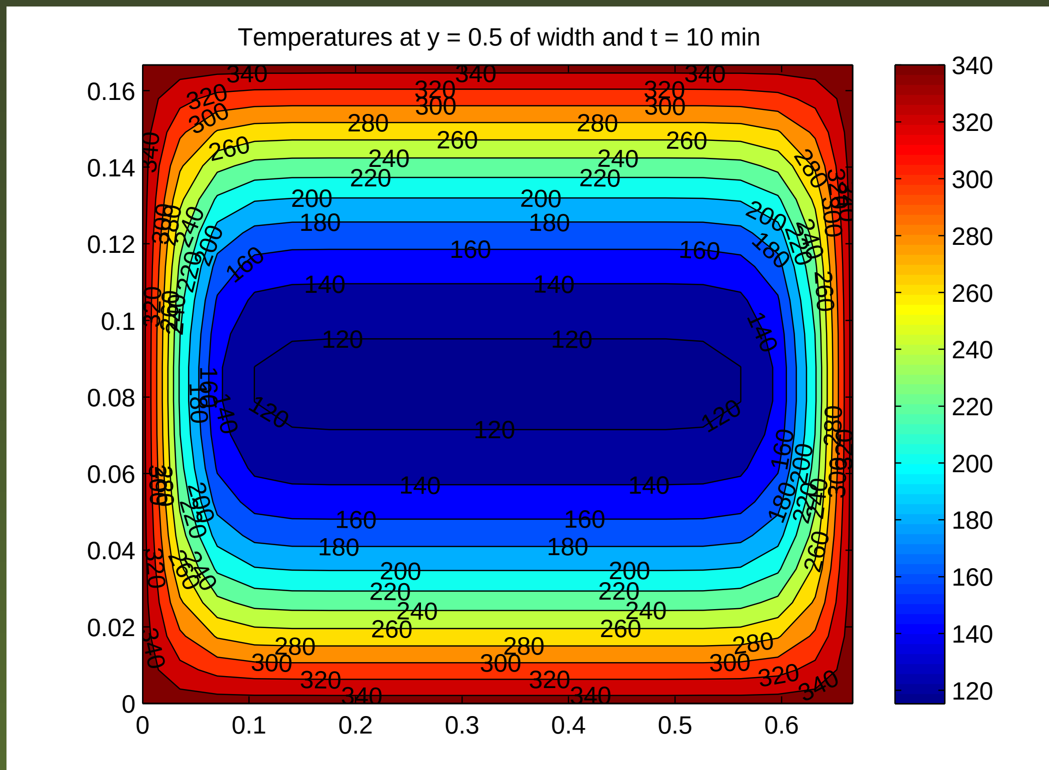

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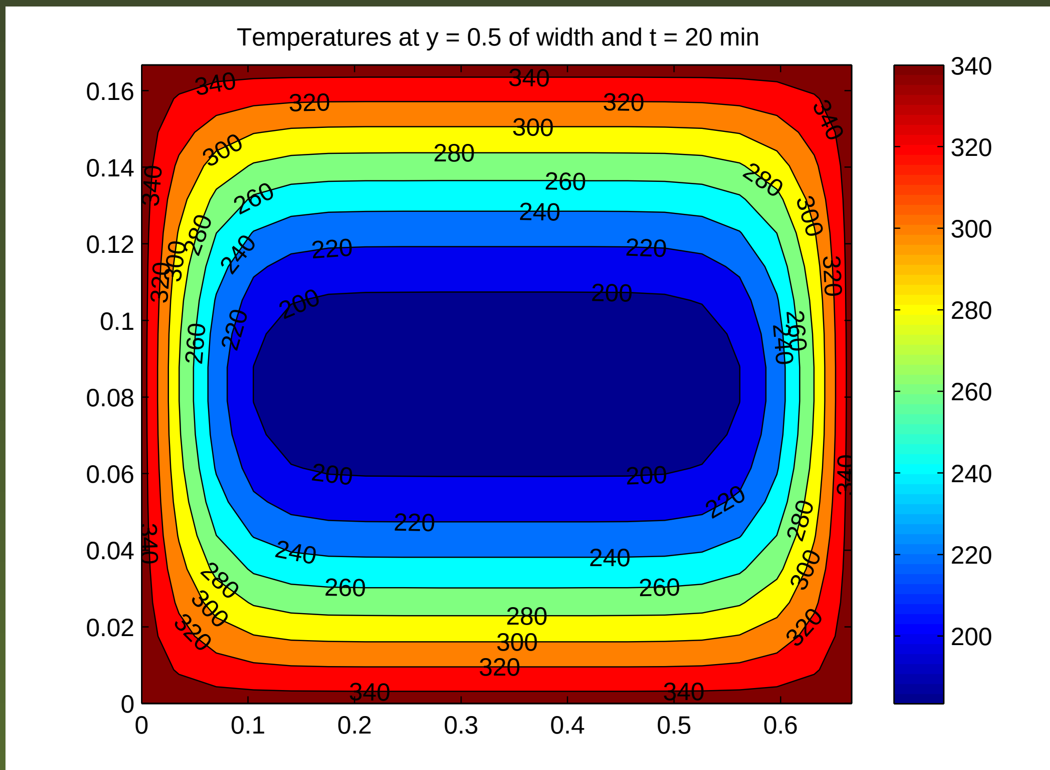

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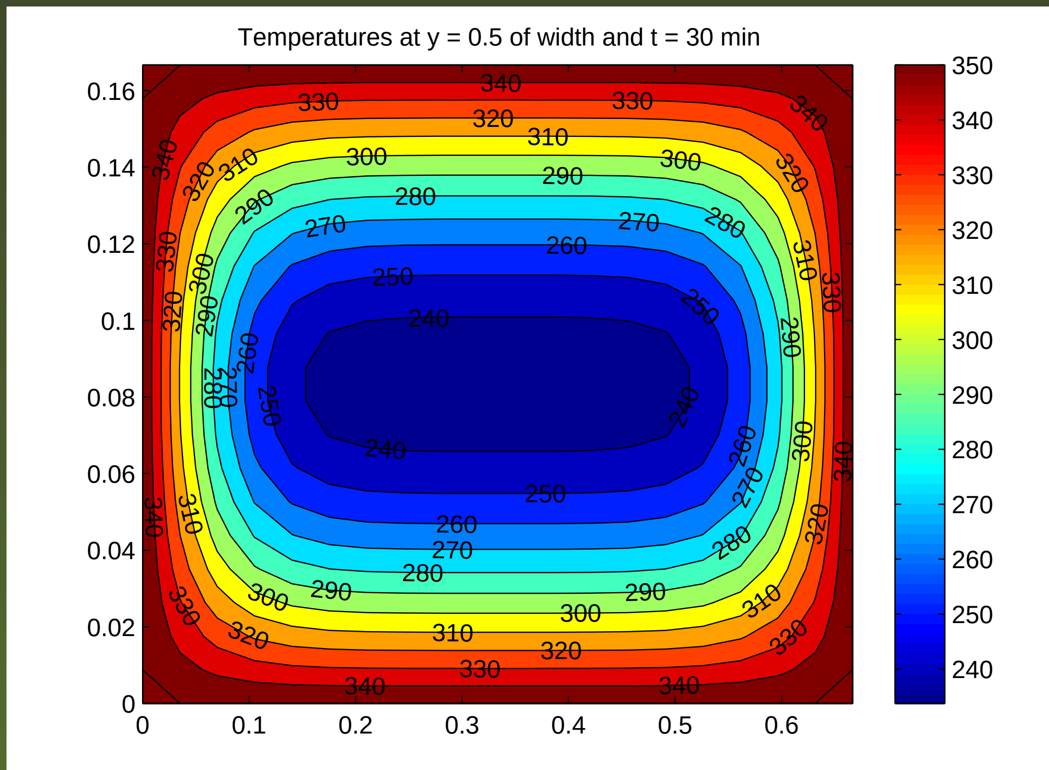

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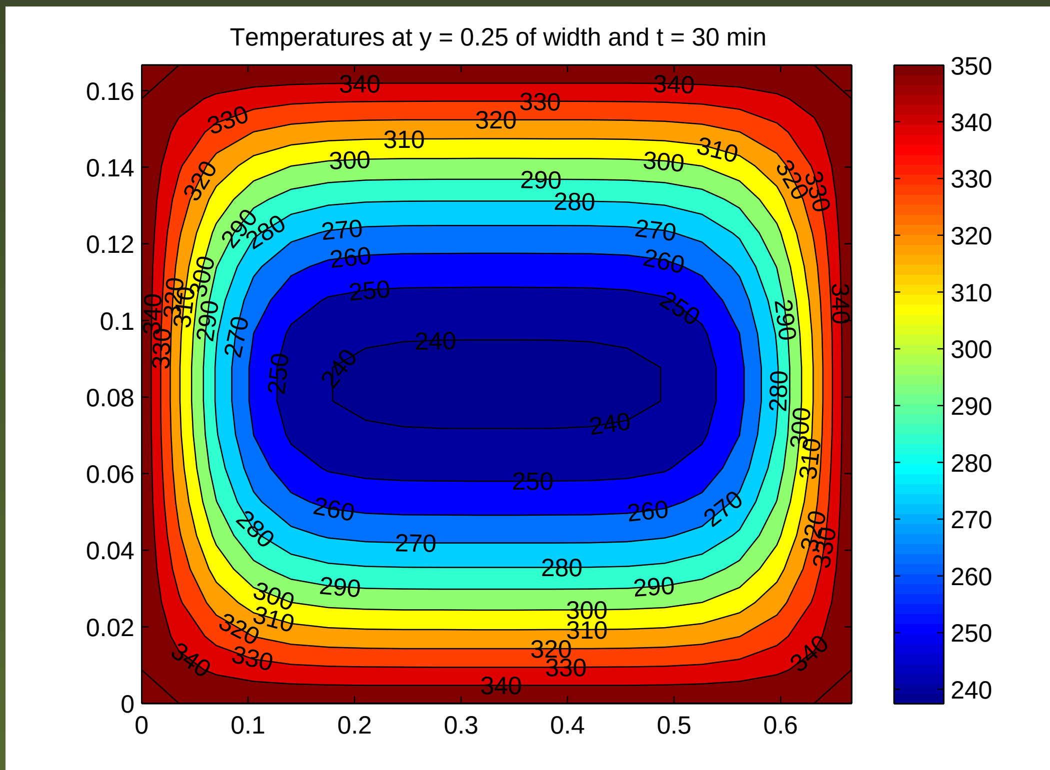

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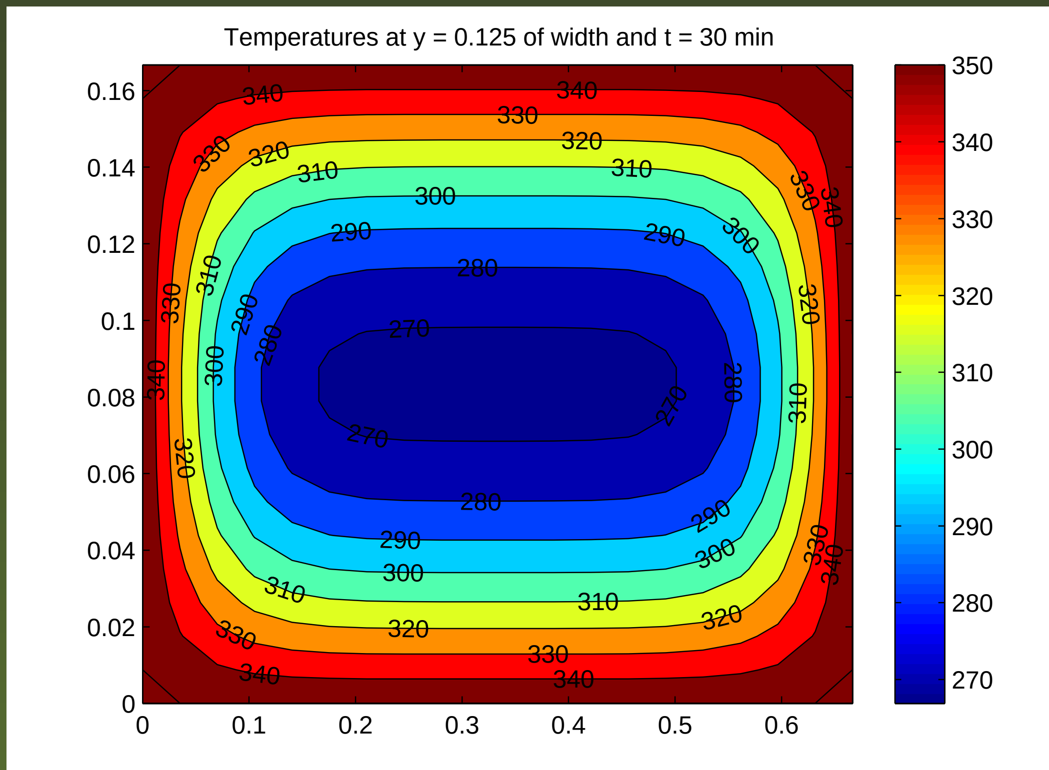

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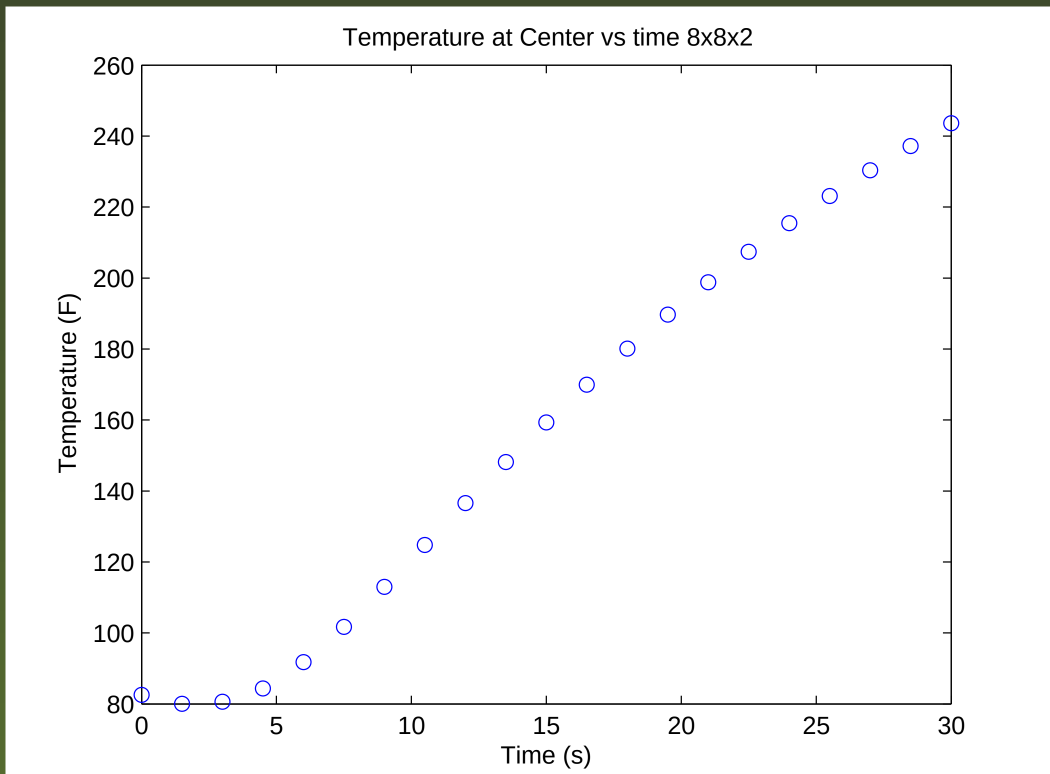
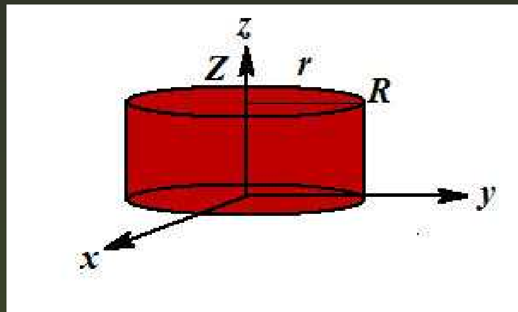


Cylindrical Cakes



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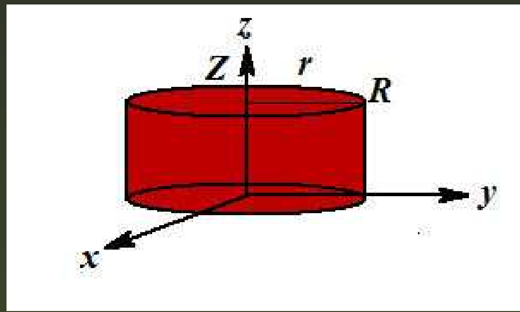

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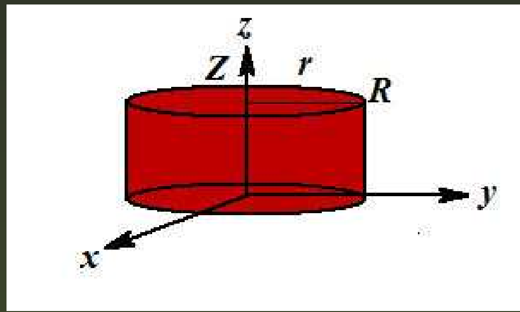

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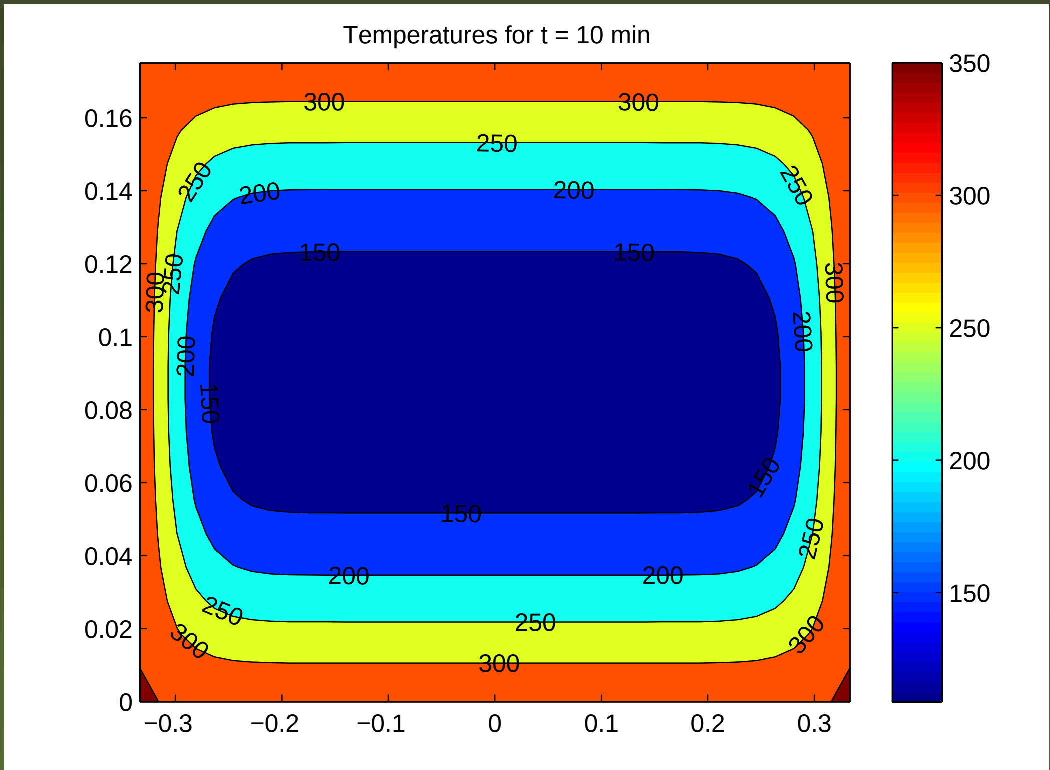

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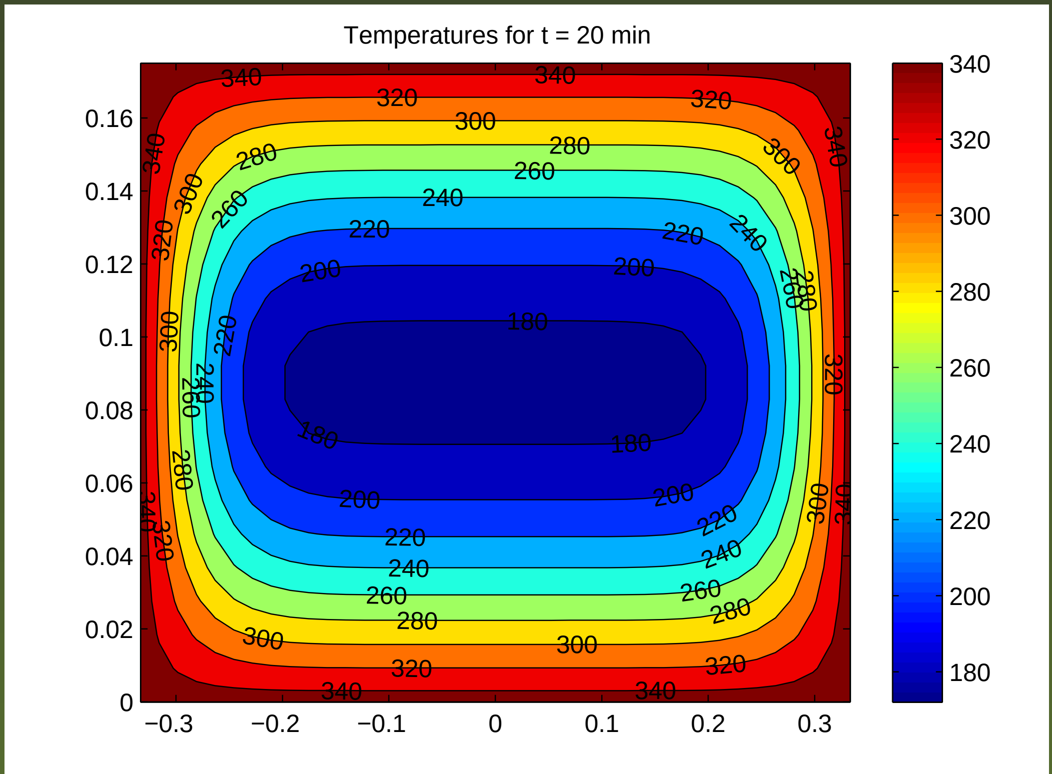

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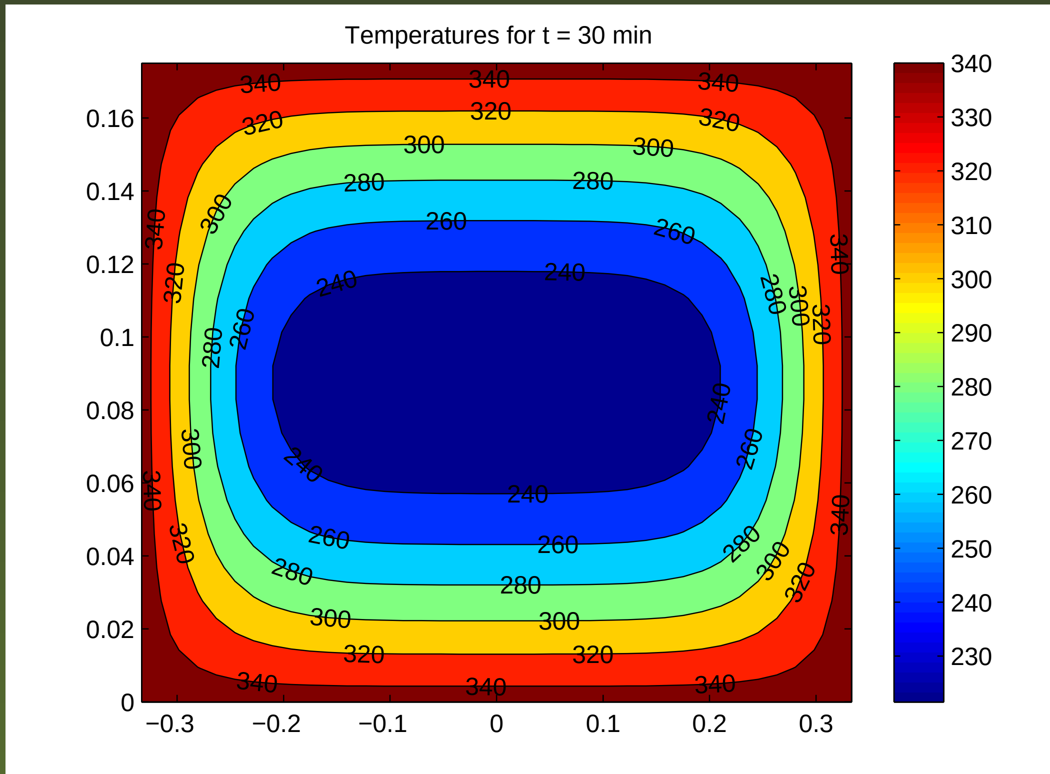

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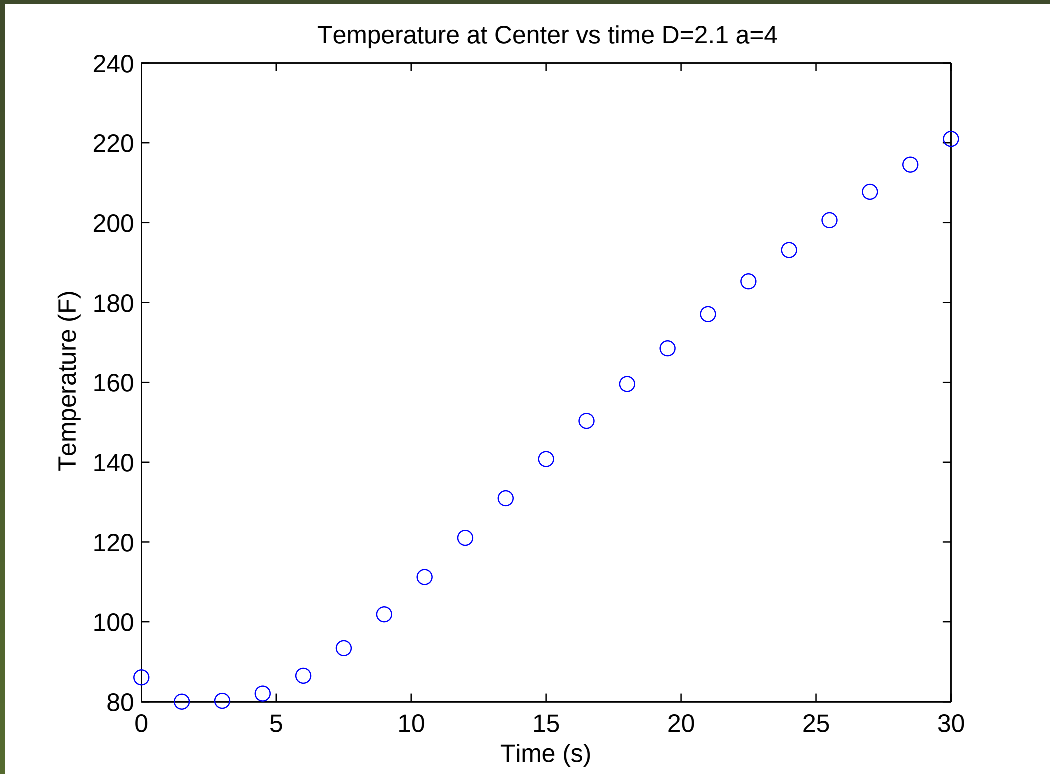

Is the Cake Done?

How long does it take to bake a cake?

- Cake is done when center is 203^oF.

Is the Cake Done?

How long does it take to bake a cake?

- Cake is done when center is 203^oF.
- Solution depends on diffusion constant.
  - Determine constant D from one cake.
  - Look at cake data and determine D.

Génoise Cake

The recipe - Jacques Pépin

Ingredient	Recipe	Dry Measure (g)
Eggs	6 large	298
Sugar	3/4 cu	176
Vanilla Extract	1/2 tsp	2
Flour	1cu	144
Butter	3/4 stick	114

Olszewski, *American Journal of Physics*, June 2006

t_f	Diam (in)	Depth (in)
17	8.0	1.0
26	4.1	4.0
20	4.1	2.0
41	9.9	1.8
35	13.0	1.6
20	9.9	1.0
19	4.1	1.8

Final Temperature

Approximate Solution (Olszewski, *AJP*, June 2006)

$$T(t_f, 0, \frac{H}{2}) = T_f \approx T_b + (T_i - T_b) * F$$

$$F \equiv \underbrace{\frac{2}{j_{01} J_1(j_{01})} e^{-\left(\frac{j_{01}}{a}\right)^2 D t_f}}_{C_1} \underbrace{\frac{4}{\pi} e^{-\left(\frac{\pi}{H}\right)^2 D t_f}}_{C_2}$$

$$\frac{1}{t_f} \propto \begin{cases} \frac{j_{01}^2}{a^2} + \frac{\pi^2}{H^2}, & C_1, C_2 \leq 1 \\ \frac{\pi^2}{H^2}, & C_1 > 1 \\ \frac{j_{01}^2}{a^2}, & C_2 > 1 \end{cases}$$

UNCW

Baking Times and Diffusion

Diam (in)	Depth (in)	t_f	Theory (1 term)	Theory (100 terms)	D (ft^2/min)
8.0	1.0	17	15	13.6	0.75597e-4
4.1	1.8	19	24	22.9	1.14186e-4
4.1	2.0	20	26	24.5	1.16302e-4
9.9	1.0	20	15	13.6	0.64379e-4
4.1	4.0	26	37	27.8	1.01339e-4
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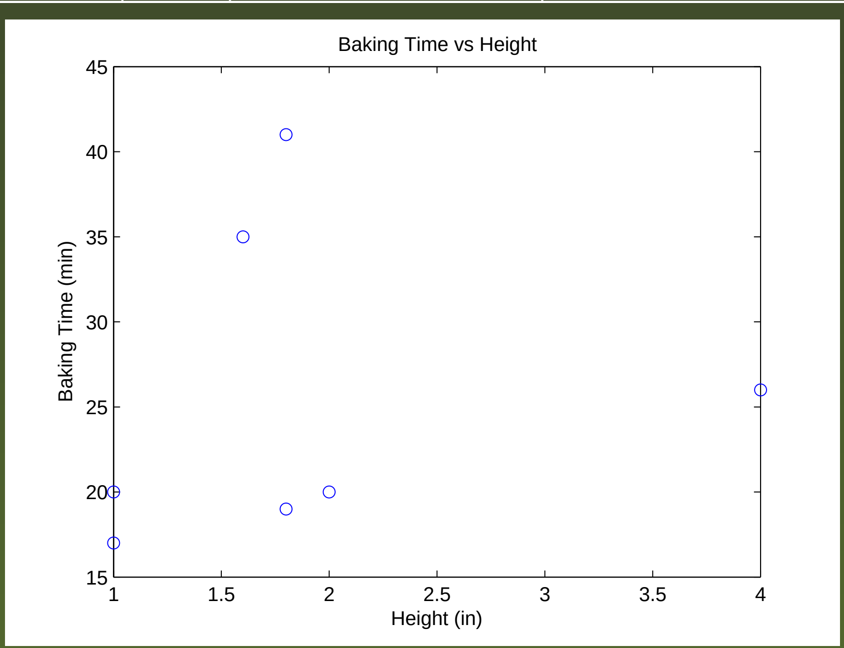

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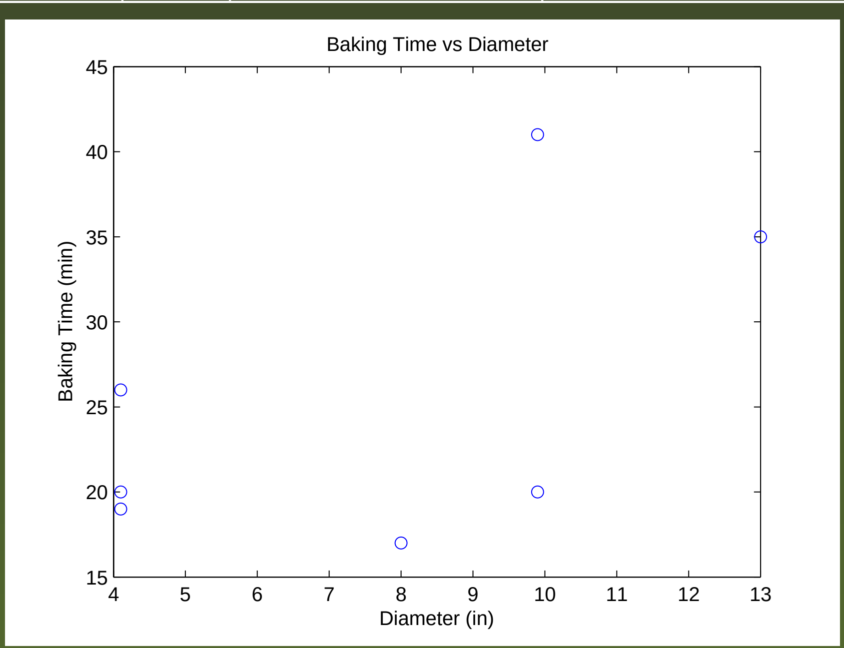

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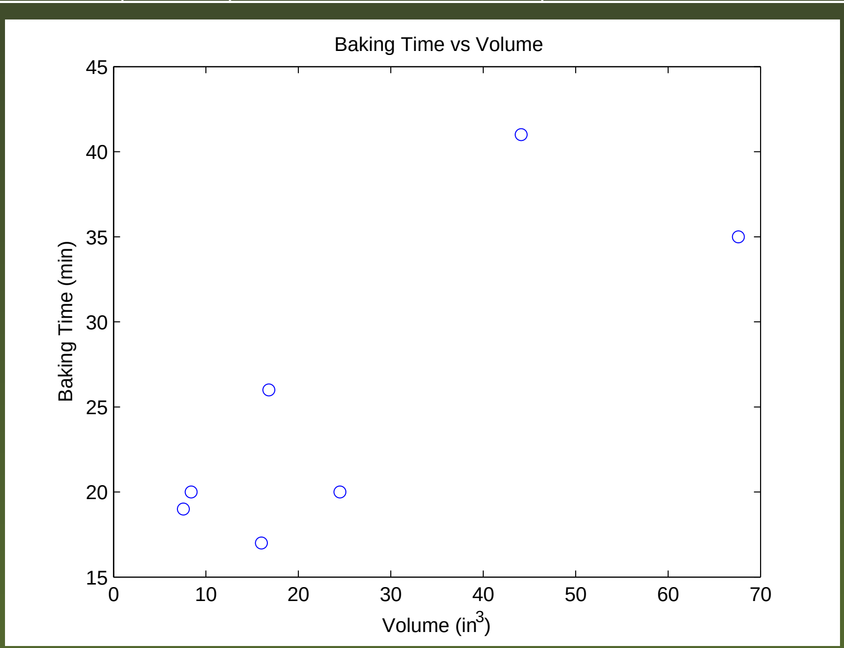

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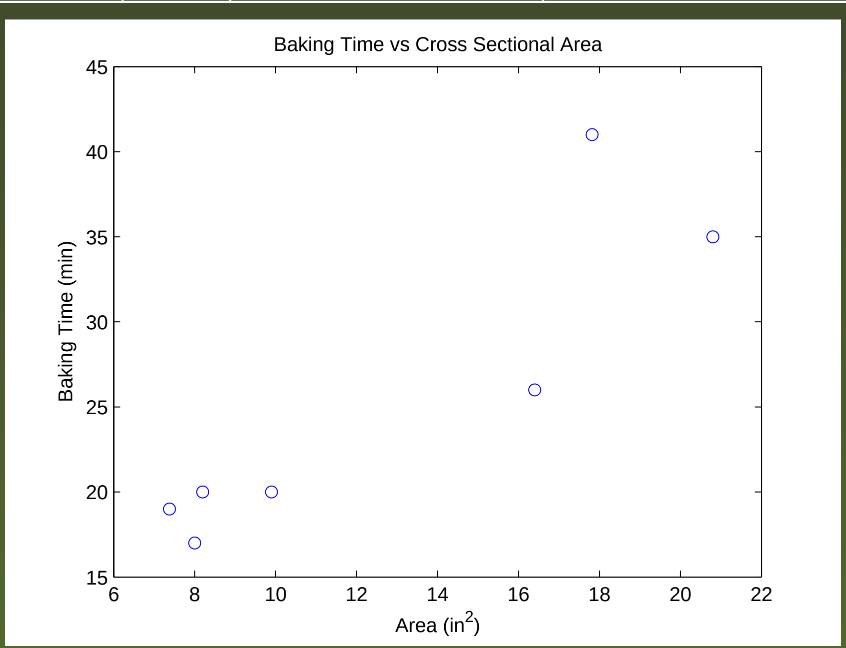

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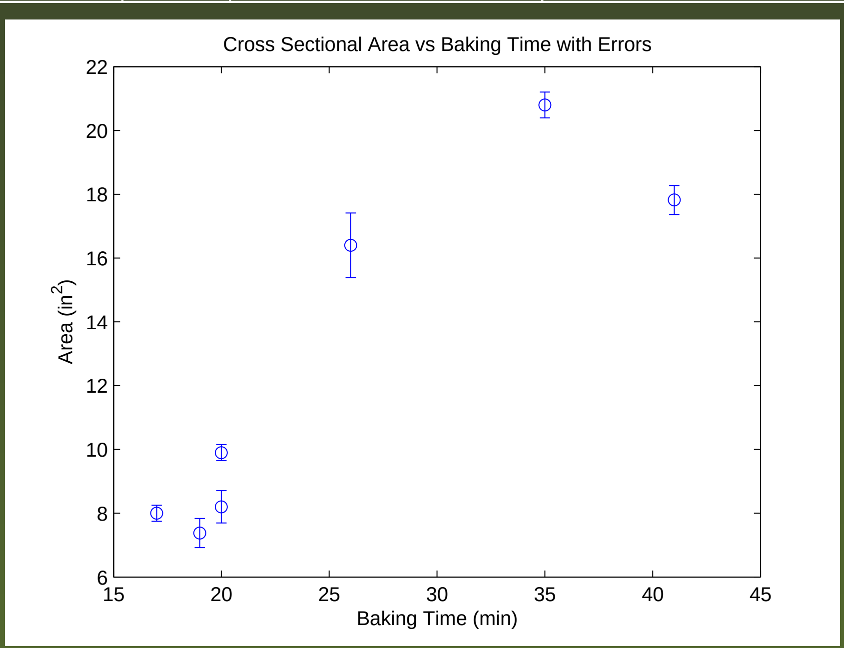

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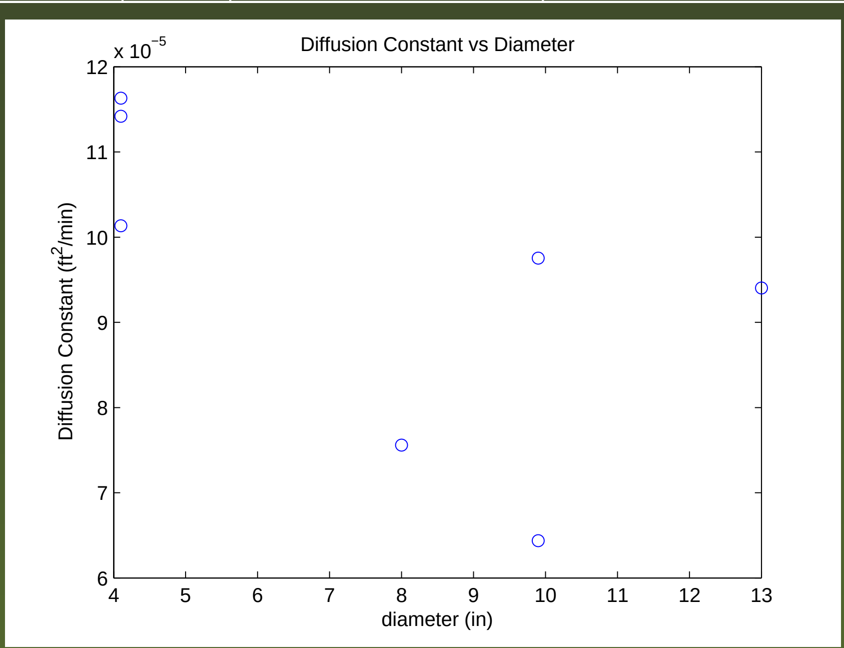

Mixed Boundary Condition

$\frac{dT}{dz} = h(T_b - T) + q = -hT + Q$ at $z = H$.

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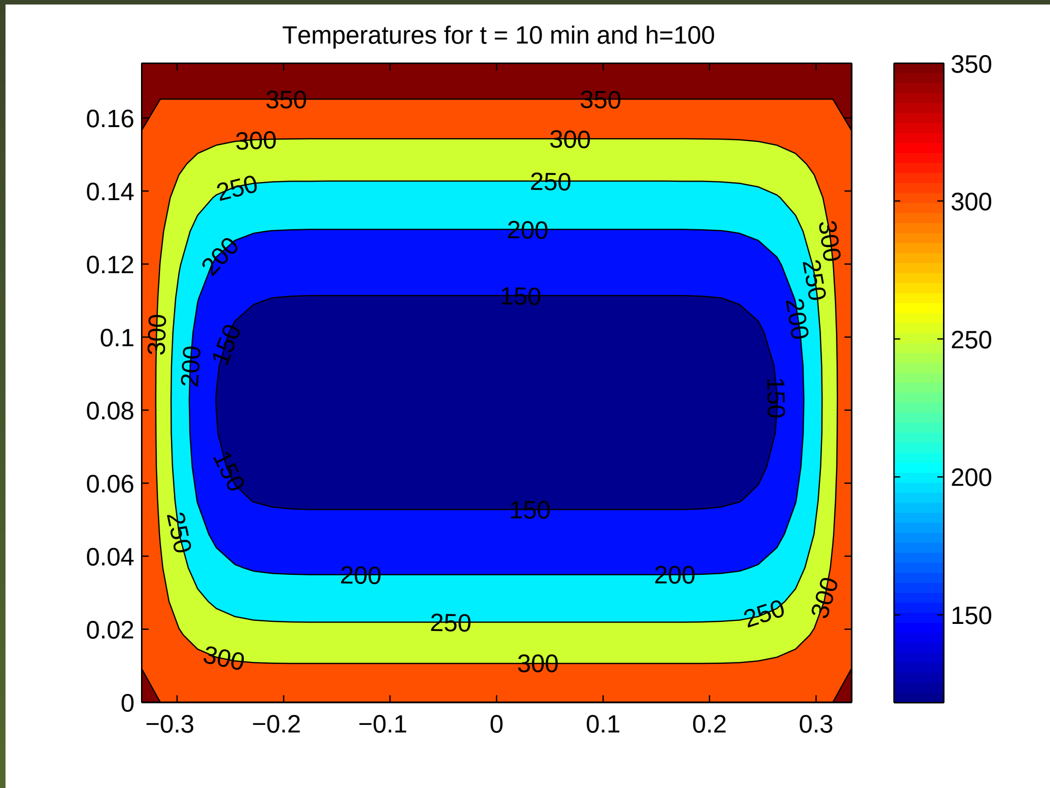

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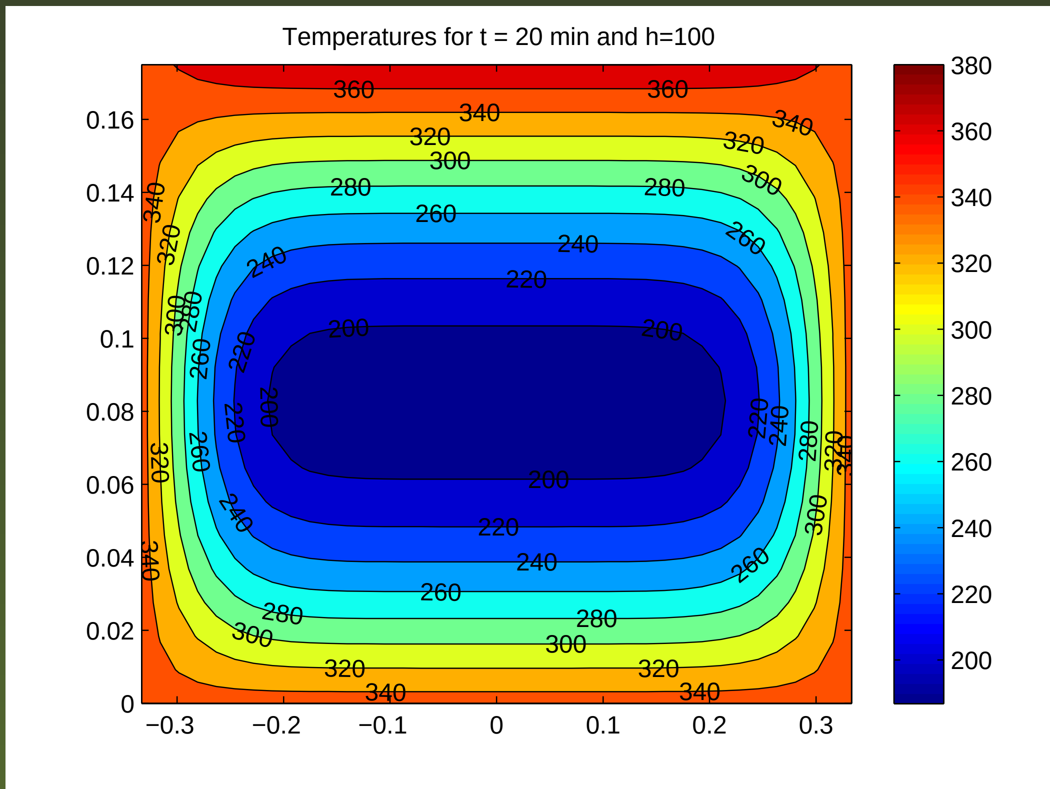

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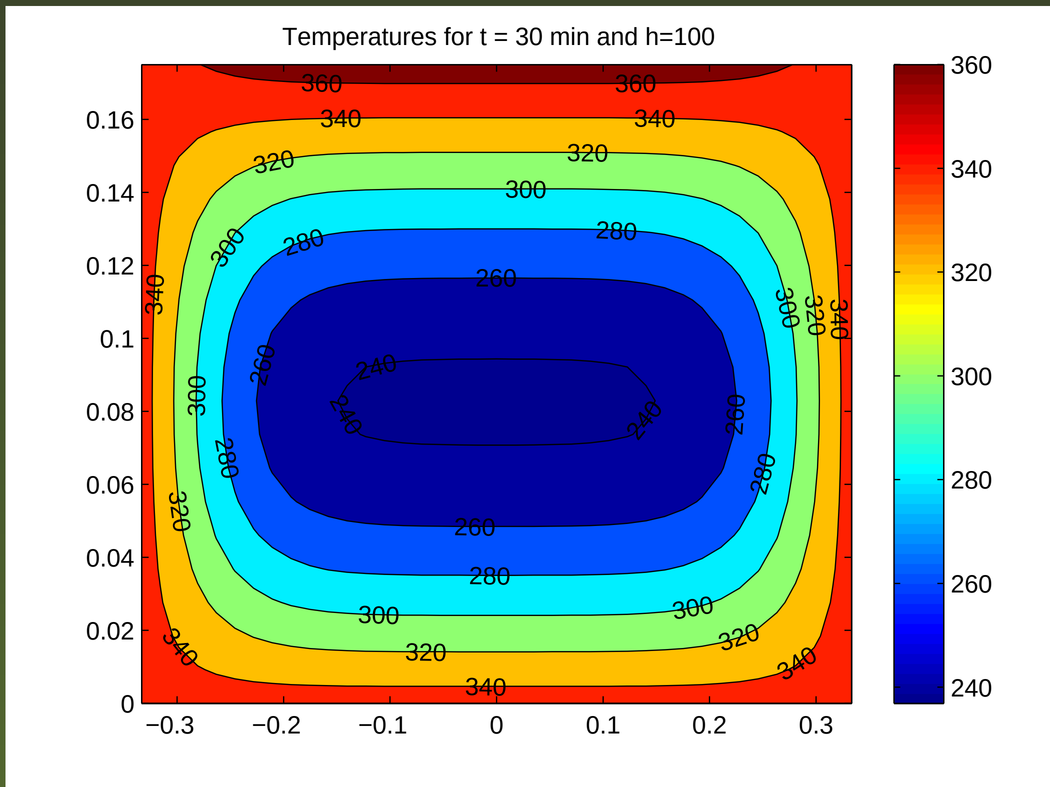

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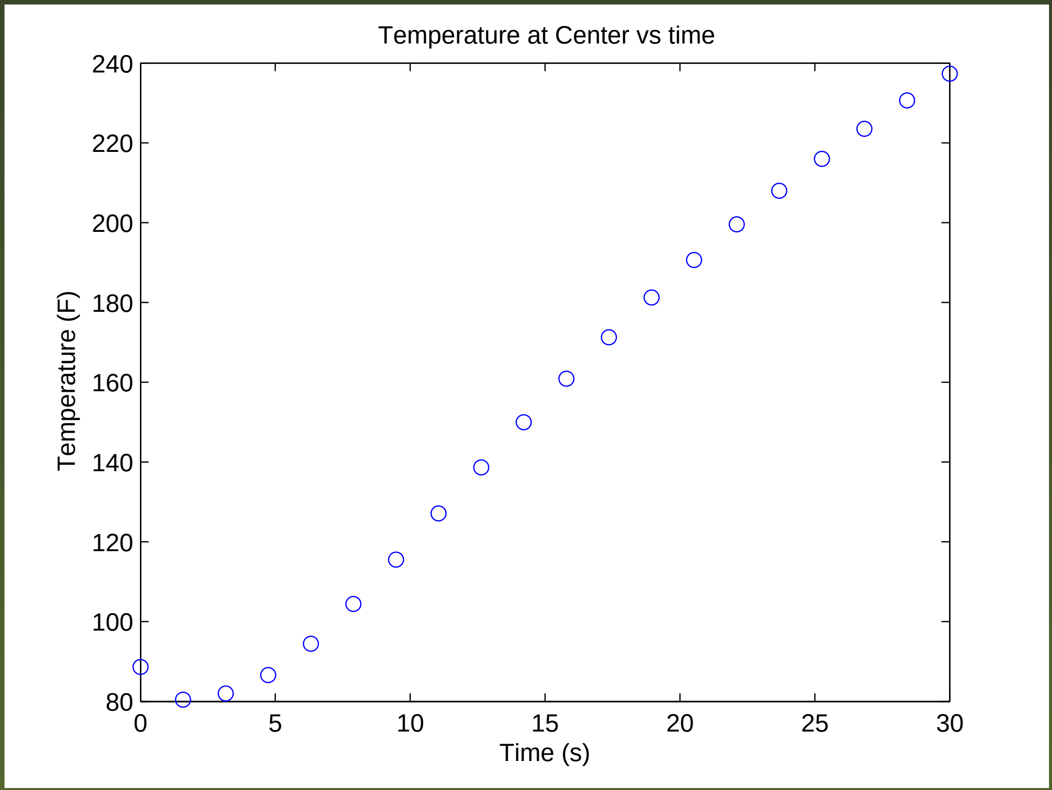

More Realistic Models

- Volume increases, mass decreases

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- Parameters change during baking process
  - conductivity
  - density
  - specific heat
  - heat diffusivity
  - convection coefficients

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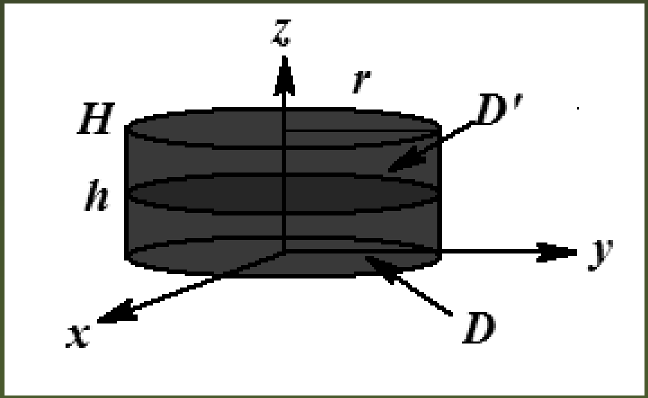

Eigenfunctions

New eigenfunctions

$$\Psi_{mj}(r,z,t) = J_0(\frac{j_{01m}}{a}r)Z_{mj}(z)e^{-\lambda t}$$

where

$$Z_{mj}(z) = \begin{cases} \sin\mu z, & 0 \le z < h, \\ \sin\mu'(H-z), & h < z \le H. \end{cases}$$

Assume $Z(z)$ and $D(z)\frac{\partial Z}{\partial z}$ are continuous

Coupled PDE Models

- Most Models for Bread Baking

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- Boundary Conditions
  - Conduction to dough
  - Convection from air
  - Radiation from oven walls

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Boundary Conditions, Bottom Surface

$$\frac{\partial u}{\partial x} = 0, \qquad T = T_1(\tau)$$

References

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