From Solitary Waves to Rogue Waves
Solutions of Nonlinear Evolution Equations

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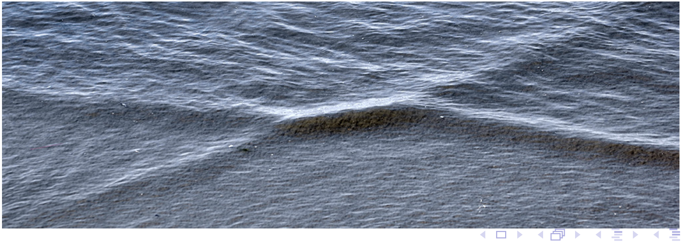


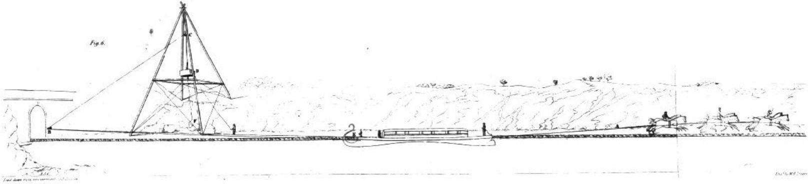
Outline

- History of Solitary Waves
- Rise of Nonlinear Wave Research
- Rogue waves
- Emergence of Rogue Wave Research

Small one meter high rogue wave in Baltic Sea, 2007 formed from 30 cm waves.

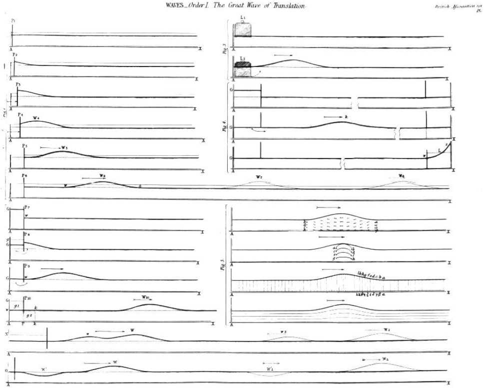
Great Wave of Translation - 1834

I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stoppednot so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour [14km/h], preserving its original figure some thirty feet [9m] long and a foot to a foot and a half [300-450mm] in height. Its height gradually diminished, and after a chase of one or two miles [23km] I lost it in the windings of the channel. Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation. John Scott Russell



Union Canal, Hermiston, Scotland, `http://www.ma.hw.ac.uk/~chris/canal.jpg`

John Scott Russell (1808-1882)



- Engineer
- Edinburgh
- Used 30 tank
- $v^2 = g(h + a)$

`http://www.ma.hw.ac.uk/~chris/scott_russell.html`

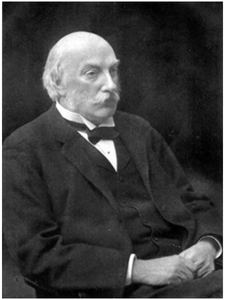
50 Years of Controversy

George Biddle Airy (1801-1892).

Sir George Gabriel Stokes (1819-1903).

1870s Nonlinear Theory/Solution

Joseph Valentin Boussinesq (1842-1929).



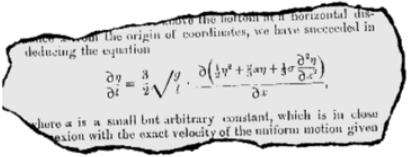
John William Strutt (Lord Rayleigh) (1842-1919).

$$u(x,t) = 2\eta^2 \operatorname{sech}^2(\eta(x - 4\eta^2 t)).$$

Korteweg-de Vries Equation - 1895

Gustav de Vries (1866-1934)

Diederik Johannes Korteweg (1848-1941)

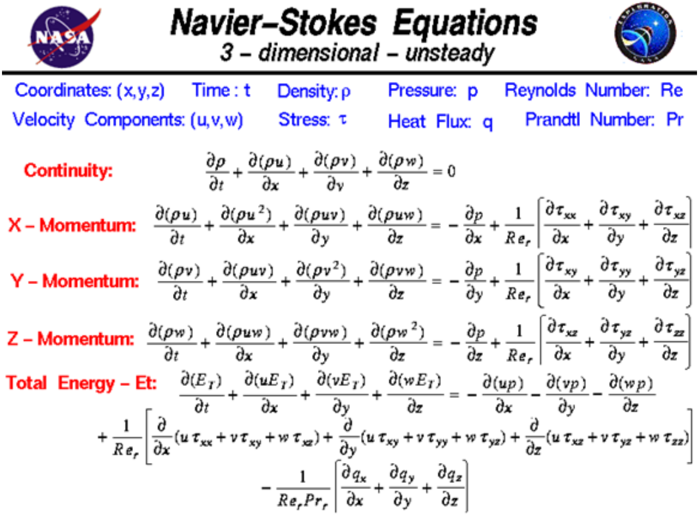


above the bottom at a horizontal dis-
the origin of coordinates, we have succeeded in
deducing the equation

$$\frac{\partial \eta}{\partial t} = \frac{3}{2}\sqrt{\frac{g}{l}} \cdot \frac{\partial\left(\frac{1}{2}\eta^2 + \frac{2}{3}\alpha\eta + \frac{1}{3}\sigma\frac{\partial^2 \eta}{\partial x^2}\right)}{\partial x},$$

here α is a small but arbitrary constant, which is in close
exion with the exact velocity of the uniform motion given

Fluid Equations



Navier–Stokes Equations

3 – dimensional – unsteady

Coordinates: (x,y,z)	Time : t	Density: ρ	Pressure: p	Reynolds Number: Re
Velocity Components: (u,v,w)		Stress: τ	Heat Flux: q	Prandtl Number: Pr

Continuity:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

X – Momentum:

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \frac{\partial(\rho uw)}{\partial z} = -\frac{\partial p}{\partial x} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right]$$

Y – Momentum:

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} + \frac{\partial(\rho vw)}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right]$$

Z – Momentum:

$$\frac{\partial(\rho w)}{\partial t} + \frac{\partial(\rho uw)}{\partial x} + \frac{\partial(\rho vw)}{\partial y} + \frac{\partial(\rho w^2)}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right]$$

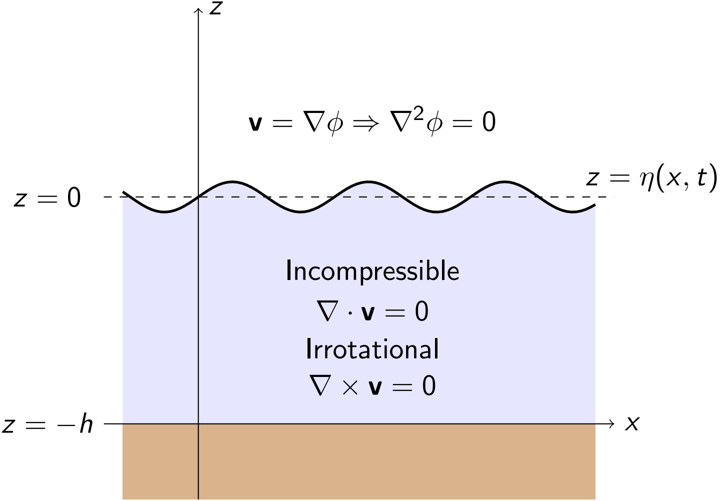
Total Energy – Et:

$$\frac{\partial(E_T)}{\partial t} + \frac{\partial(uE_T)}{\partial x} + \frac{\partial(vE_T)}{\partial y} + \frac{\partial(wE_T)}{\partial z} = -\frac{\partial(up)}{\partial x} - \frac{\partial(vp)}{\partial y} - \frac{\partial(wp)}{\partial z}$$

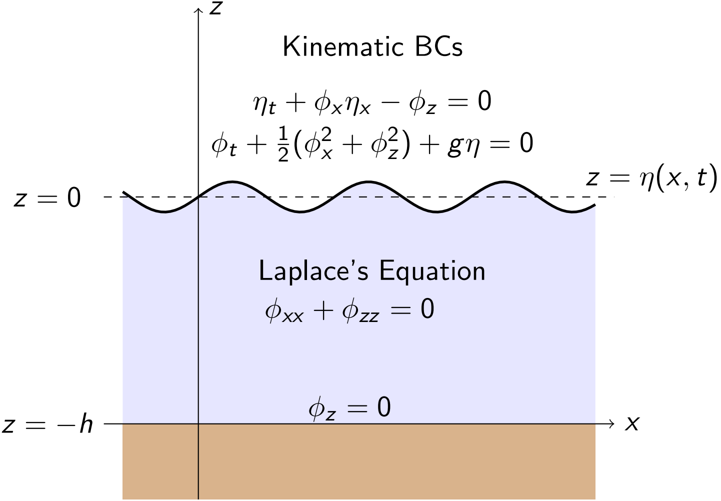
$$+\frac{1}{Re_r}\left[\frac{\partial}{\partial x}(u\tau_{xx} + v\tau_{xy} + w\tau_{xz}) + \frac{\partial}{\partial y}(u\tau_{xy} + v\tau_{yy} + w\tau_{yz}) + \frac{\partial}{\partial z}(u\tau_{xz} + v\tau_{yz} + w\tau_{zz})\right]$$

$$-\frac{1}{Re_r Pr_r}\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right]$$

Derivation -Laplace's Equation



Derivation - Perturbation Theory



Kinematic BCs

$$\eta_t + \phi_x \eta_x - \phi_z = 0$$

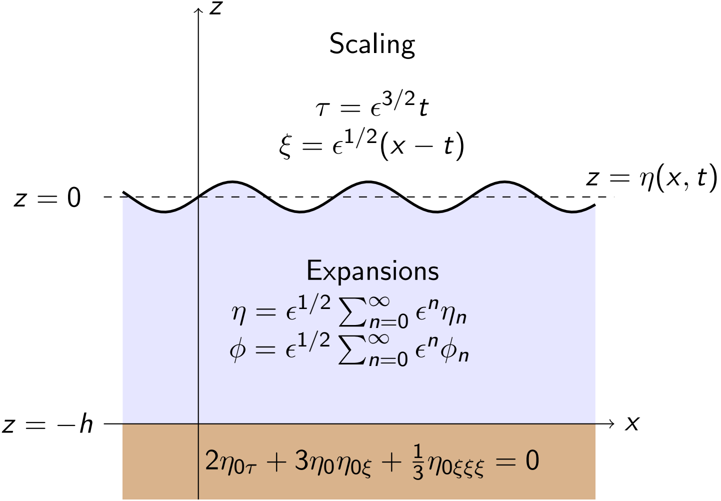
$$\phi_t + \frac{1}{2}(\phi_x^2 + \phi_z^2) + g\eta = 0$$

Laplace's Equation

$$\phi_{xx} + \phi_{zz} = 0$$

$$\phi_z = 0$$

Derivation - KdV Equation



Scaling

$$\tau = \epsilon^{3/2} t$$
$$\xi = \epsilon^{1/2}(x - t)$$

Expansions

$$\eta = \epsilon^{1/2} \sum_{n=0}^{\infty} \epsilon^n \eta_n$$
$$\phi = \epsilon^{1/2} \sum_{n=0}^{\infty} \epsilon^n \phi_n$$

$$2\eta_{0\tau} + 3\eta_0\eta_{0\xi} + \frac{1}{3}\eta_{0\xi\xi\xi} = 0$$

KdV Traveling Waves - $u_t + 6uu_x + u_{xxx} = 0$

Seek solutions $u(x,t) = f(\xi)$, where $\xi = x - ct$. Using $u_x = f'(\xi)$ and $u_t = -cf'(\xi)$, we have

$$-cf' + 6ff' + f''' = 0.$$

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$$-cf + 3f^2 + f'' = A.$$

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$$-cff' + 3f^2f' + f''f' = Af'.$$

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$$-\frac{1}{2}cf^2 + f^3 + \frac{1}{2}f'^2 = Af + B.$$

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$$\frac{c}{2}f'^2 = Af + B - (-\frac{1}{2}cf^2 + f^3).$$

KdV Traveling Waves - $u_t + 6uu_x + u_{xxx} = 0$

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$$\tfrac{c}{2}f'^2 = Af + B - (-\tfrac{1}{2}cf^2 + f^3).$$

$$\sqrt{\frac{c}{2}}\frac{df}{d\xi} = \sqrt{Af + B + \frac{1}{2}cf^2 - f^3}.$$

KdV Traveling Waves - $u_t + 6uu_x + u_{xxx} = 0$

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$$\xi - \xi_0 = \sqrt{\frac{c}{2}}\int \frac{df}{\sqrt{Af + B + \frac{1}{2}cf^2 - f^3}}.$$

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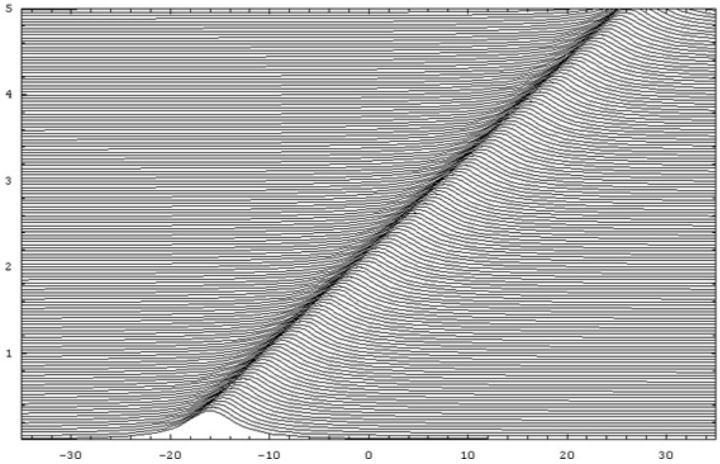
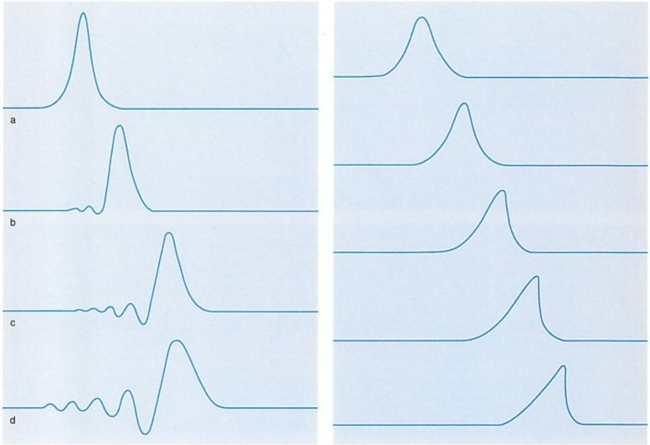
$u(x,t) = 2\eta^2 \operatorname{sech}^2(\eta(x - 4\eta^2 t))$ where $c = 4\eta^2$.

Soliton Solution - $u(x,t) = 2\eta^2 \operatorname{sech}^2(\eta(x - 4\eta^2 t))$



Time vs position plot of soliton solution, for the KdV $u_t + 6uu_x + u_{xxx} = 0$.

Dispersion vs Nonlinearity - $u_t + 6uu_x + u_{xxx} = 0$



Dispersion (left): Waves spread and amplitude diminishes.

Nonlinearity (right): Width decreases and waves steepen. (Herman 1992)

Soliton Experiment

Water tank `http://www.youtube.com/watch?v=SknvLa8qEu0&feature=related`

Re-enactment

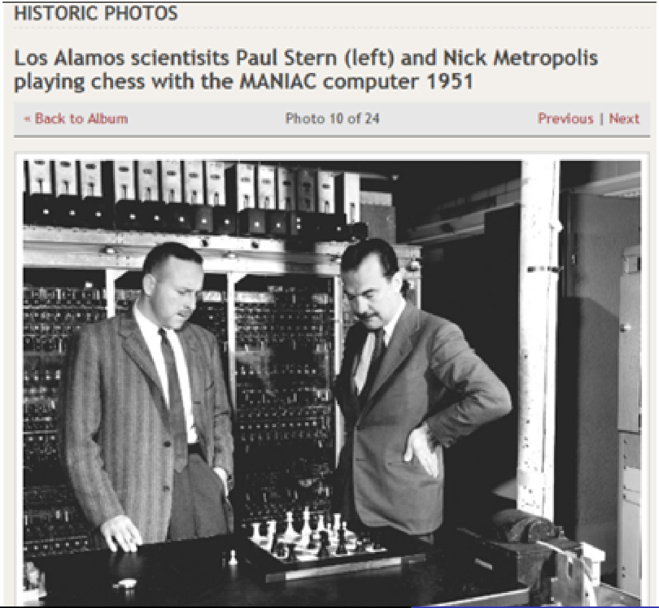
July 12, 1995 Union Canal, Scott Russell Aqueduct,

`http://apachepersonal.miun.se/~tomnil/solitoner/solipic.htm`

Severn Bore - England

`http://captainkaisworld.blogspot.com/2011/04/surfing-rivers-7-ghosts-severn-chinas.html`

The Age of Computing

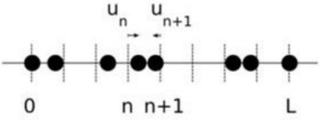


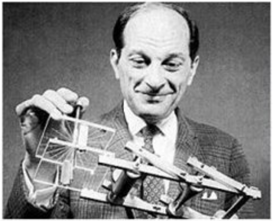
HISTORIC PHOTOS

Los Alamos scientisits Paul Stern (left) and Nick Metropolis playing chess with the MANIAC computer 1951

Fermi, Pasta, Ulam (FPU) Problem (1953-4)

- Simulate 1D chain of masses linked by nonlinear springs.
- Linear - energy of vibration modes remains in the mode
- Nonlinear - allows energy transfer between modes.





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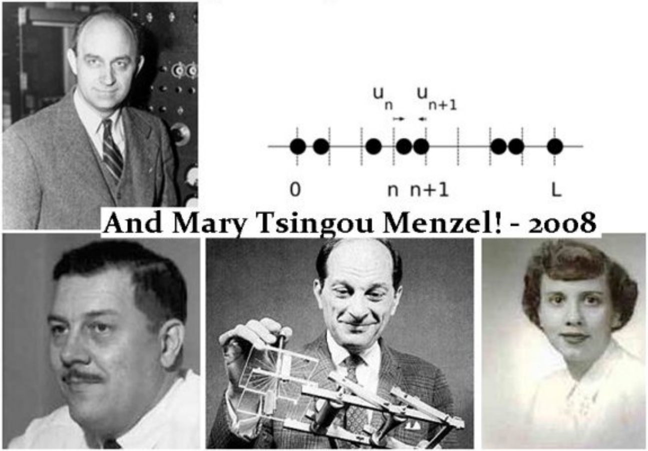
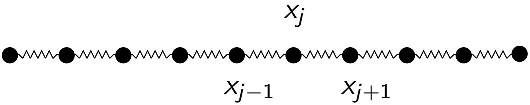


And Mary Tsingou Menzel! - 2008

FPU Problem



$$\begin{aligned} m\ddot{x}_j &= k(x_{j+1} - 2x_j + x_{j-1})[1 + \alpha(x_{j+1} - x_{j-1})] \\ \ddot{x}_j &= \frac{c^2}{h^2}(x_{j+1} - 2x_j + x_{j-1})[1 + \alpha(x_{j+1} - x_{j-1})], \end{aligned}$$

where $c^2 = \frac{Y}{\rho}$, $\rho = m/h^3$.

- Surprising behavior: The energy does not drift toward the equipartition predicted by statistical physics but periodically returns to the original mode.
- Los Alamos 1955 report - beginning of nonlinear physics and the age of computer simulations

From FPU to KdV

Let $x_{j+1}(t) = u(x+h,t)$, $x_j(t) = u(x,t)$, $x_{j-1}(t) = u(x-h,t)$:
Use Taylor expansions:

$$\begin{aligned} \frac{x_{j+1} - 2x_j + x_{j-1}}{h^2} &= u_{xx} + \frac{h^2}{12} u_{xxxx} + O(h^4) \qquad (1) \\ \alpha(x_{j+1} - x_{j-1}) &= 2\alpha h u_x + \frac{\alpha h^3}{3} u_{xxx} + O(h^5), \qquad (2) \end{aligned}$$

Then,

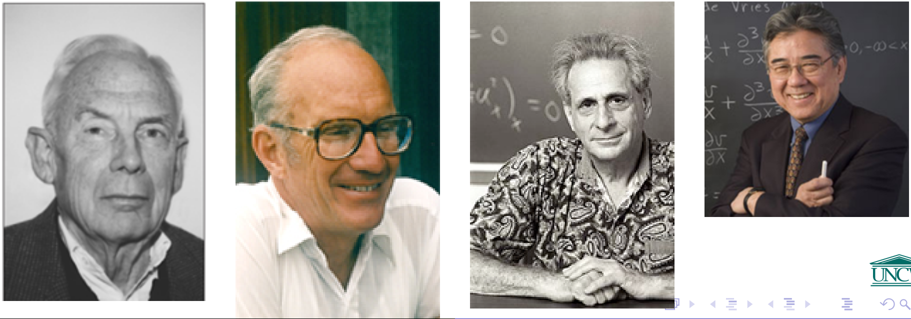
$$\frac{1}{c^2} u_{tt} - u_{xx} \approx 2\alpha h u_x u_{xx} + \frac{h^2}{12} u_{xxxx}$$

Let $\xi = x - ct$, $\tau = (\alpha ch)t$, $y(\xi,\tau) = u(x,t)$,

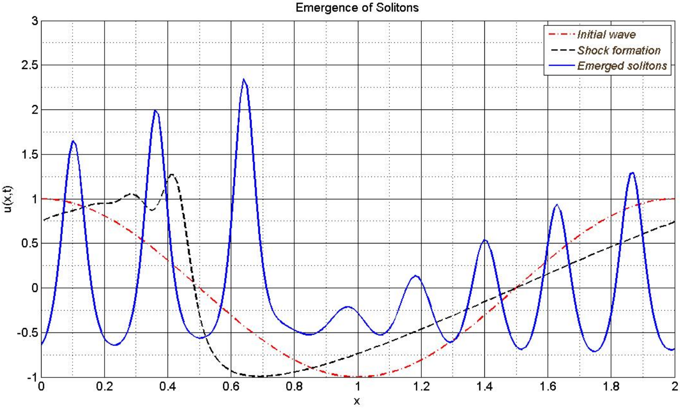
$$\begin{aligned} 0 &= y_{\xi\tau} + y_\xi y_{\xi\xi} - \delta^2 y_{\xi\xi\xi\xi} \\ 0 &= v_\tau + v v_\xi - \delta^2 v_{\xi\xi\xi}. \qquad (3) \end{aligned}$$

The KdV Resurgence 1960s

- Kruskal and Zabusky 1965
    - FPU Problem → KdV
    - Coined term "soliton"
- Gardner, Greene, Kruskal, Miura 1967
    - Inverse Scattering Transform
    - NLEE revolution



ZK - Recurrence



Soliton Research Begins

- IST - Nonlinear Fourier Transform

$$\begin{tikzcd}
u(x,0) \arrow[r, "\textit{Transform}"] \arrow[d, dashed, "u_t + 6uu_x + u_{xxx} = 0"'] & A(k,0) \arrow[d, "A_t = ik^3A"'] \\
u(x,t) & A(k,t) \arrow[l, "\textit{Inverse Scattering Transform}"]
\end{tikzcd}$$

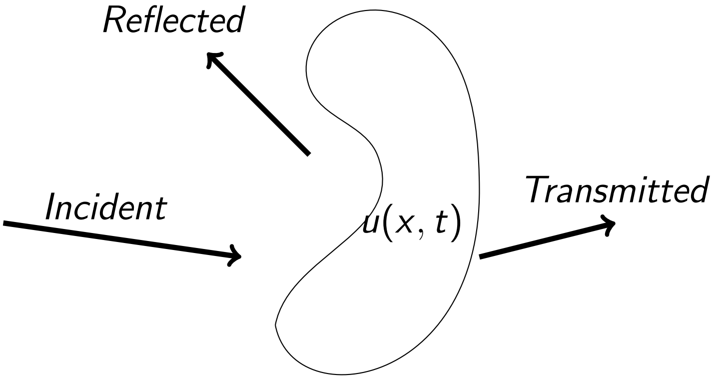
- Solutions of nonlinear PDEs
  - Lax - 1968 $L\phi = \lambda\phi, \phi_t = B\phi \Rightarrow L_t = [L, B]$.
  - Zakharov and Shabat - 1973
  - Ablowitz, Kaup, Newell, Segur - 1974
  - Research extended to other equations, dimensions
  - Other methods for finding exact solutions, N-soliton solutions

IST for KdV

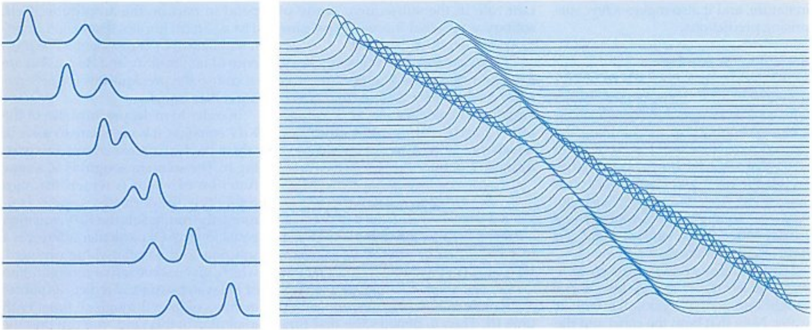
$$L\phi = -\phi_{xx} + u(x,t)\phi = \lambda\phi$$

$$\phi_t = B\phi = u_x\phi + (4\lambda - 2u)\phi_x$$

$$\Rightarrow u_t = [L, B] = -uu_x - u_{xxx}.$$



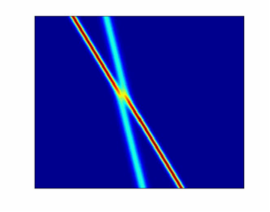
Two Soliton Solution of KdV

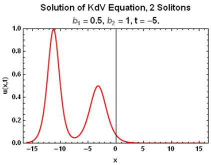


$$u(x,t) = 12\frac{3 + 4\cosh(2\xi + 24t) + \cosh(4\xi)}{[3\cosh(\xi - 12t) + \cosh(3\xi + 12t)]^2},$$

where $\xi = x - 16t$.

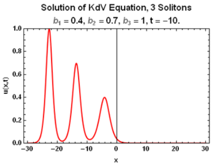
Solitons Elastic Collisions





Solution of KdV Equation, 2 Solitons

$b_1 = 0.5$, $b_2 = 1$, t = −5.



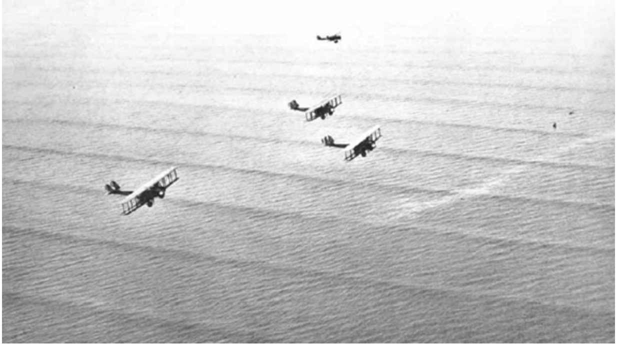
Solution of KdV Equation, 3 Solitons

$b_1 = 0.4$, $b_2 = 0.7$, $b_3 = 1$, t = −10.

`http://www.scholarpedia.org/article/Soliton`

Cnoidal Waves

$\eta(x,t) = \eta_2 + H\text{cn}^2\left(\frac{K(m)}{\lambda}(x-ct)|m\right)$ where $cn(x|m)$ is Jacobi elliptic function and $K(m)$ is complete elliptic integral.

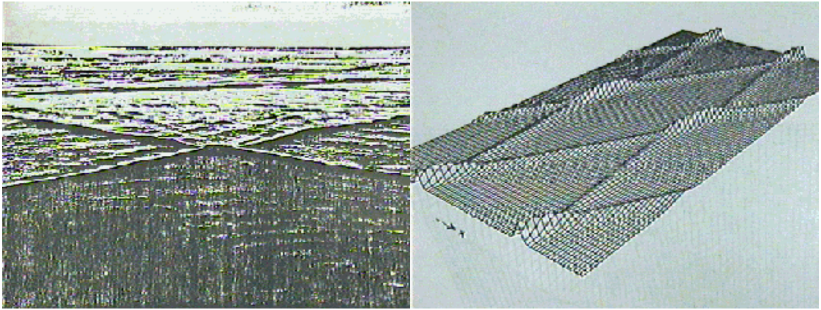


US Army bombers flying close to the Panama coast (1933).

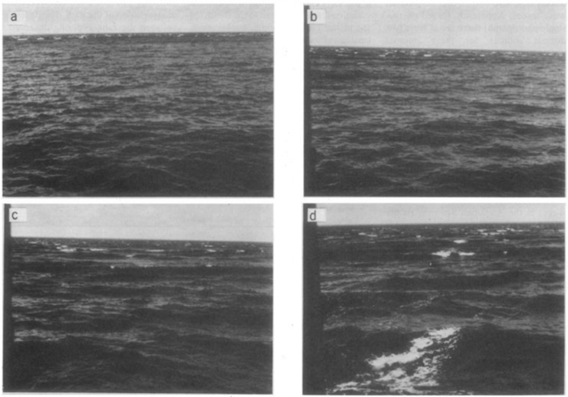
Two-Dimensional Waves

KadomtsevPetviashvili equation (1970),

$$(u_t + uu_x + u_{xxx})_x + u_{yy} = 0.$$



Internal Waves



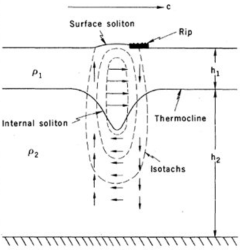
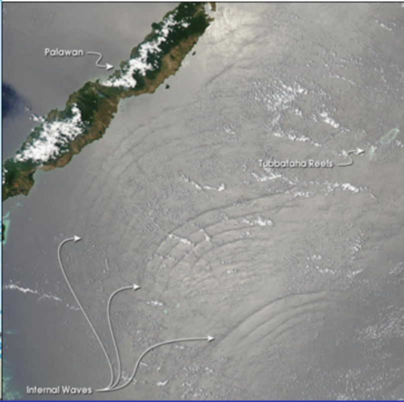


Fig. 3. An internal soliton in a two-layer fluid of finite depth is a wave of depression when $h_1 < h_2$. Dashed lines are lines of constant water particle speed (isotachs); the arrows indicate the magnitude and direction of the water particles. A small surface soliton of amplitude $\sim (\rho_2 - \rho_1)\eta_0$ accompanies the internal soliton (*35*). The approximate location of the surface rips, as observed in our Andaman Sea measurements, is also shown.

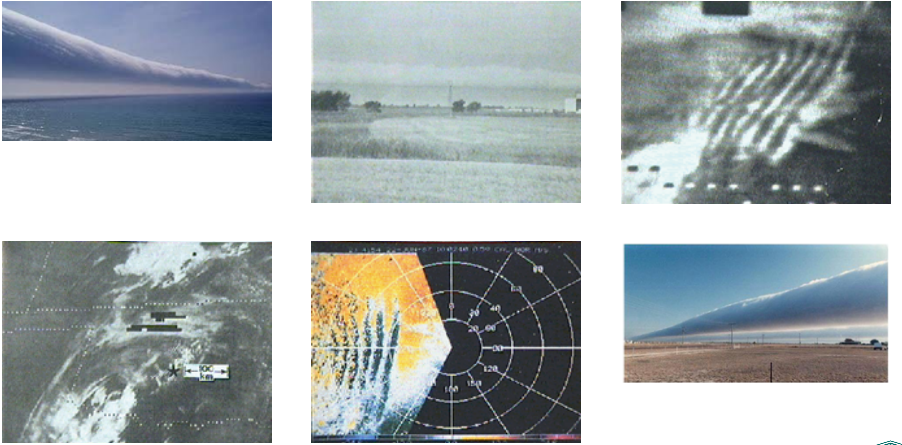
Rip Waves, Osborne and Burch - *Science*, Vol. 208, No. 4443 (May 2, 1980), pp. 451-460

Sulu Sea

www.middlebury.edu/~scs



Atmospheric Solitons



Optical Solitons

- 1973 Hasegawa, Tappert predicted optical solitons in communications
- 1987, First experimental observation in an optical fiber.
- 1988, Mollenauer, et al. transmitted pulses 4,000 km.
- 1991, Bell Labs - transmitted solitons error-free, 2.5 Gb, >14,000 km.
- 1998, Georges, et al. - data transmission of 1 Tb/s (10^{12} bits of information per sec).
- 2001, Algety Telecom deployed submarine telecom equipment in Europe, carrying real information using John Scott Russell's solitary wave.

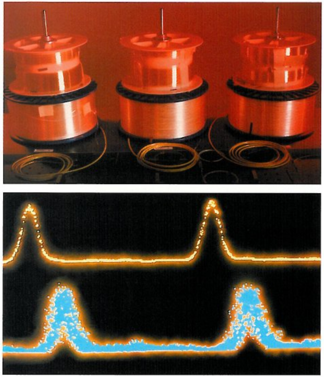


Figure 10. Fiber-optic communications could be enhanced by encoding information in solitons. Investigators at AT&T Bell Laboratories have been experimenting with soliton propagation since they first transmitted one through an optical fiber in 1980. Here the oscilloscope trace of a series of solitons is shown before *(yellow trace)* and after *(blue trace)* traveling 10,000 kilometers in an optical fiber. The pulses show little tendency to dispersion. The transmission rate was five billion bits per second, which would be equivalent to about 100,000 digitized voice channels. (Photograph courtesy of AT&T Bell Laboratories.)

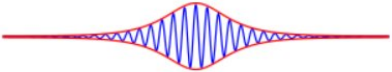
NLS Equation

The nonlinear Schrödinger equation:

$$i\frac{\partial\psi}{\partial t} = \frac{1}{2}\frac{\partial^2\psi}{\partial x^2} + \psi|\psi|^2.$$

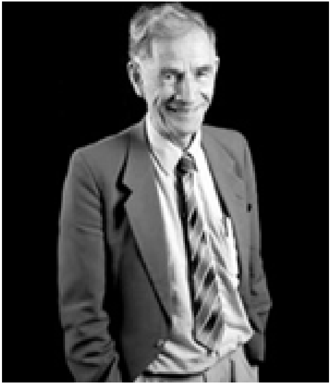
For water waves, $\eta(x,t) = a(x,t)\cos(kx - \omega t - \theta)$, where $\psi = ae^{i\theta}$, or $a = |\psi|$.

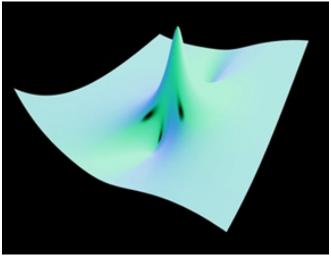
$$\psi(x,t) = 2\beta e^{2\alpha(x+4\alpha t-4(\alpha^2+\beta^2))}\,\text{sech}(2\beta(x+4\alpha t+\delta))$$



Modulational wave solution of NLS.

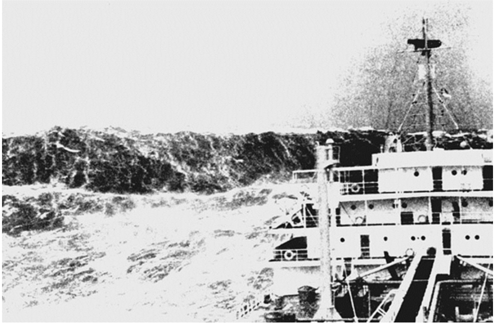
Peregrine Soliton





- Howell Peregrine (1938-2007)
- 1983 Peregrine predicted spatio-temporal evolution of an NLS soliton
- 20 years later used as prototypical example of rogue waves in water and in optics.

Bay of Biscay, France - 1940



Merchant ship laboring in heavy seas as a huge wave looms astern. Huge waves are common near the 100-fathom line in the Bay of Biscay. Published in Fall 1993 issue of Mariner's Weather Log. http://en.wikipedia.org/wiki/File:Wea00800,1.jpg

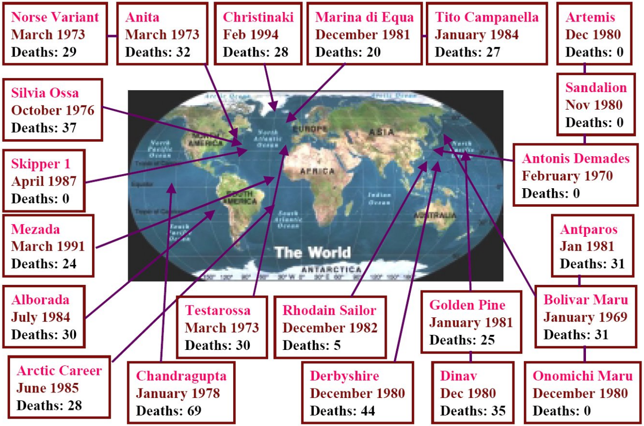
Bering Sea - 1979

Ship Discoverer gets pounded by monster wave in the Bering Sea.

Charleston, SC - 1991

The Perfect Storm - `http://www.opc.ncep.noaa.gov/perfectstorm/index.shtml`

Rogue Wave Incidents



Norse Variant
March 1973
Deaths: 29

Anita
March 1973
Deaths: 32

Christinaki
Feb 1994
Deaths: 28

Marina di Equa
December 1981
Deaths: 20

Tito Campanella
January 1984
Deaths: 27

Artemis
Dec 1980
Deaths: 0

Silvia Ossa
October 1976
Deaths: 37

Sandalion
Nov 1980
Deaths: 0

Skipper 1
April 1987
Deaths: 0

Antonis Demades
February 1970
Deaths: 0

Mezada
March 1991
Deaths: 24

Antparos
Jan 1981
Deaths: 31

Alborada
July 1984
Deaths: 30

Testarossa
March 1973
Deaths: 30

Rhodain Sailor
December 1982
Deaths: 5

Golden Pine
January 1981
Deaths: 25

Bolivar Maru
January 1969
Deaths: 31

Arctic Career
June 1985
Deaths: 28

Chandragupta
January 1978
Deaths: 69

Derbyshire
December 1980
Deaths: 44

Dinav
Dec 1980
Deaths: 35

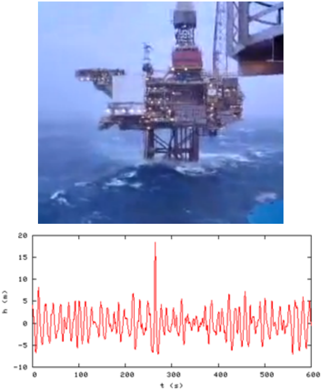
Onomichi Maru
December 1980
Deaths: 0

Reported collisions with rogue waves:

`http://hal.archives-ouvertes.fr/docs/00/00/03/52/PDF/Rogue_wave_V1.pdf`

Draupner Wave Jan 1, 1995

- Oil platform in the central North Sea
- Minor damage
- Read by a laser sensor.
- During wave heights of 12 m (39ft),
  - Freak wave - max height of 25.6 m (84ft)
  - (peak elevation was 18.5 m (61ft)).



Oil platform and time series.

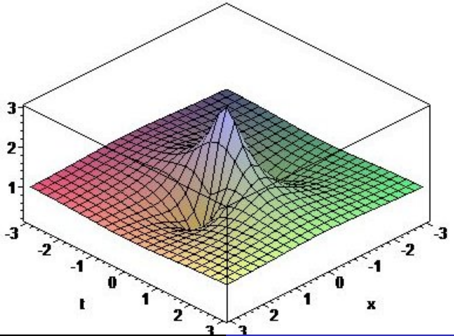
Rational Solutions

NLS Equation,

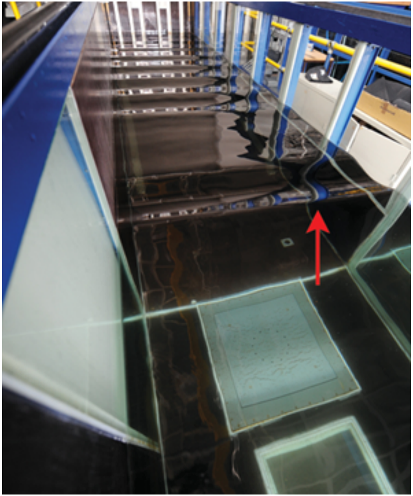
$$i\frac{\partial\psi}{\partial t}+\frac{1}{2}\frac{\partial^2\psi}{\partial x^2}+\psi|\psi|^2=0.$$

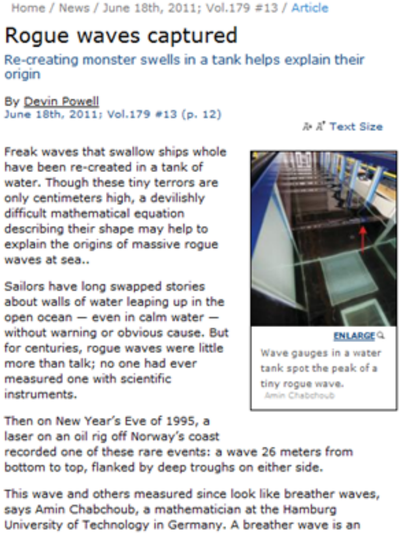
Peregrine solution,

$$\psi(x,t)=\left(1-\frac{4(1+2it)}{1+4x^2+4t^2}\right)e^{it}$$



Rogue Waves 2010





Home / News / June 18th, 2011; Vol.179 #13 / Article

Rogue waves captured

Re-creating monster swells in a tank helps explain their origin

By Devin Powell
June 18th, 2011; Vol.179 #13 (p. 12)

Freak waves that swallow ships whole have been re-created in a tank of water. Though these tiny terrors are only centimeters high, a devilishly difficult mathematical equation describing their shape may help to explain the origins of massive rogue waves at sea..

Sailors have long swapped stories about walls of water leaping up in the open ocean — even in calm water — without warning or obvious cause. But for centuries, rogue waves were little more than talk; no one had ever measured one with scientific instruments.

Wave gauges in a water tank spot the peak of a tiny rogue wave.
Amin Chabchoub

Then on New Year's Eve of 1995, a laser on an oil rig off Norway's coast recorded one of these rare events: a wave 26 meters from bottom to top, flanked by deep troughs on either side.

This wave and others measured since look like breather waves, says Amin Chabchoub, a mathematician at the Hamburg University of Technology in Germany. A breather wave is an

The Research Continues

2784 A. Ankiewicz et al. / *Physics Letters A* 375 (2011) 2782–2785

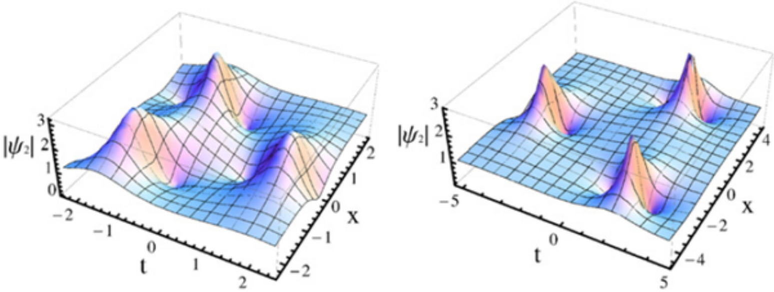
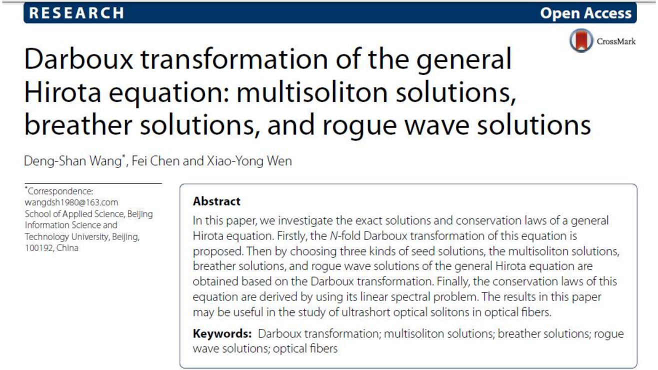


Fig. 3. Rogue wave triplets. Parameters (a) $\gamma = 20$ and $\beta = 40$; (b) $\gamma = 100$ and $\beta = -400$.

Current Research



RESEARCH
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Darboux transformation of the general Hirota equation: multisoliton solutions, breather solutions, and rogue wave solutions

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School of Applied Science, Beijing Information Science and Technology University, Beijing, 100192, China

Abstract

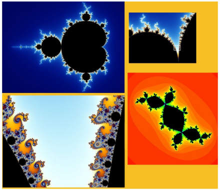
In this paper, we investigate the exact solutions and conservation laws of a general Hirota equation. Firstly, the *N*-fold Darboux transformation of this equation is proposed. Then by choosing three kinds of seed solutions, the multisoliton solutions, breather solutions, and rogue wave solutions of the general Hirota equation are obtained based on the Darboux transformation. Finally, the conservation laws of this equation are derived by using its linear spectral problem. The results in this paper may be useful in the study of ultrashort optical solitons in optical fibers.

Keywords: Darboux transformation; multisoliton solutions; breather solutions; rogue wave solutions; optical fibers

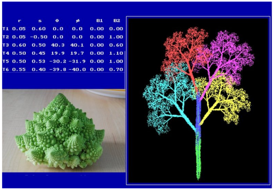
Summary

- History of integrable PDEs (KdV, NLS, etc)
  - Solitons, Kinks, Breathers, Loop solitons
  - Rational solutions - rogue waves
- Solution Techniques
  - Inverse Scattering
  - Lie Symmetries
  - New Solution Methods
  - Darboux Transformations
  - Perturbation Theory
- Rogue waves exist! - 1995 Draupner data
- Active area of rogue wave research
  - Analytical - new methods of solution generation
  - Numerical - robustness of solutions
  - Experimental [optics, hydrodynamics, plasmas]

Other Nonlinear Mathematics Topics

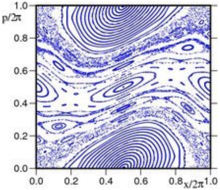


Mandelbrot and Julia sets.



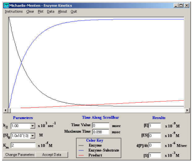
	r	s	θ	φ	B1	B2
T1	0.05	0.60	0.0	0.0	0.00	0.00
T2	0.05	-0.50	0.0	0.0	0.00	1.00
T3	0.60	0.50	40.3	40.1	0.00	0.60
T4	0.50	0.45	19.9	19.7	0.00	1.10
T5	0.50	0.53	-30.2	-31.9	0.00	1.00
T6	0.55	0.40	-39.8	-40.0	0.00	0.70

Iterated function systems.



Discrete maps and chaos.

Cardiac dynamics.



k_2 1.00 x 10^3 sec^{-1}

$[S]_0$ 1.0x10^(-3) M

K_m 2 x 10^{-5} M

Time Value 0 msec

Maximum Time 0.098 msec

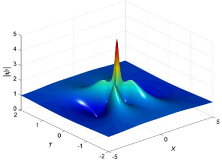
[E] 1 x 10^{-9} M

[ES] 0 x 10^{-9} M

d[P]/dt 0 x 10^{-6} M/sec

[S] 1 x 10^{-3} M

Nonlinear ODE systems.



Rogue waves.

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