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1 The One Soliton Solution Under Noise

Abstract

$$u_t + 6uu_x + u_{xxx} = \zeta(t), \quad (1)$$

We consider the one soliton solution of the KdV:

∞ ... $\int^t$ ... $dt' \Big) \Big] \Big\rangle .$

So far, we have

∞

Therefore, the transform solution is

$$u_t + 6uu_x + u_{xxx} = \zeta(t) - \gamma u. \quad (20)$$

Large Time Behavior of the Soliton Peak:

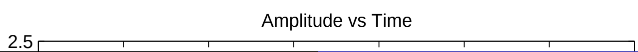
Zabusky and Kruskal (1965) studied the KdV equation using the finite difference approximation [**?**]:

We use discretized Brownian motion, where $W(t)$ is specified at discrete t values.

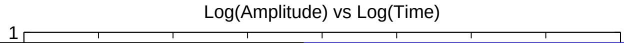
[fontsize=] randn('state',100) T = 1; N = 500; dt = T/N; dW = sqrt(dt)*randn(1,N); W = cumsum(dW);

The other identity to confirm is
$< \exp(cW(t)) >= \exp(\frac{1}{2} < W^2(t) >)$.

```
mu=sqrt(2*eps);
```


Amplitude vs Time
2.5

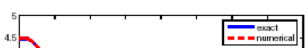
Log(Amplitude) vs Log(Time)

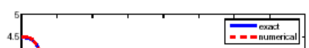


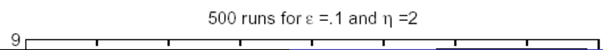
[baselinestretch=1,fontsize=] for count=1:runs for j=1:Tsteps
dW(j)=mu*sqrt(dt)*randn; end; for j=3:Tsteps for i=3:N-1

[baselinestretch=1,fontsize=]

```
A=(spdiags(-2*ones(N1),1,N1,N1)+spdiags(ones(N1),2,N1,N1)
```


5
4.5
exact
numerical


5
4.5
exact
numerical


500 runs for ε =.1 and η =2
9

$\gamma = 0.01$ $\varepsilon = 0.01$ $\gamma = 0.01$ $\varepsilon = 0.1$ $\gamma = 0.01$ $\varepsilon = 0.3$ $\gamma = 0.01$ $\varepsilon = 0.5$

= 0.1

$\gamma = 0.01$ $\varepsilon = 0.01$ $\gamma = 0.01$ $\varepsilon = 0.1$ $\gamma = 0.01$ $\varepsilon = 0.3$ $\gamma = 0.01$ $\varepsilon = 0.5$

]Decay Constants for $t \in [0, 10]$

]Decay Constants for $t \in [0, 100]$

Log-log Plot for Rescaled Integral Solution

[ε=0.1 and γ=0.1]

Log-log Plot for Rescaled Numerical Solution
[ε=0.1 and γ=0.1, 2000 Runs]

When two solitons collide, they interact elastically. The exact solution for the two soliton equation is given by

Two Soliton Solution of KdV (Zabusky-Kruskal Scheme)

Two Soliton solution for ε=.01 and γ=.01

Two Soliton solution for ε=.5 and γ=.01

1. Exact Solution of Stochastic KdV

R. L. Herman, "Solitary Waves," *American Scientist*, vol. 80, July-August 1992, 350-361.