What Should Undergraduates Know About Gravitation?

Dr. Russell Herman

Department of Mathematics & Statistics, UNCW

Abstract

In July 2006 forty five physics faculty from nearly as many universities, including one from UNCW, met at Syracuse University to discuss the importance of teaching general relativity to undergraduates. This discussion came on the heels of a year long celebration of Albert Einstein's miracle year in 1905 which marked the beginnings of the theory of relativity and the ever increasing interest in gravitation and cosmology. There were numerous talks and posters on topics that could be used to present general relativity to undergraduates. These included gravitational radiation and LIGO, deviations from Newtonian gravitation and GPS, black holes, and experimental tests of general relativity. Leading authors of new undergraduate texts on general relativity contributed their thoughts on what students can learn about relativity. One of the goals of the workshop was to produce sample syllabi for courses aimed at three different audiences: general interest, physics intensive and mathematics intensive. In this talk we will discuss some of the ideas that came out of this workshop and, of course, talk a bit about general relativity and the growing need to introduce it into the physics curriculum.

Outline

Teaching General Relativity to Undergraduates

AAPT TGRU site

http://www.aapt-doorway.org/TGRU/

The workshop was supported by the LIGO Project, the Center for Gravitational Wave Physics at Penn State, the American Association of Physics Teachers, and the Syracuse University Department of Physics.

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Information at Web Site

- Talks
- Posters
- Articles - Pedagogy/Research
- Links to Texts
- Course Design/Syllabi

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- $E = mc^2$ is more recognized than $F = ma$.

Who, What?

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  - develop math foundations (tensors and differential geometry).

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  - develop math foundations (tensors and differential geometry).
  - Einstein's Equations and then Applications.

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    - Applications first and then mathematical background.
    - Understanding gravity as curved spacetime via examples.

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Approaches - Math or Physics First?

- Math or Physics First?
- General Interest
  Possibly the only physics course.

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    - Newtonian physics - between all masses

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What Should Undergraduates Know About Gravity?

- Gravity is a universal interaction.
  - Relativistic - between all forms of energy

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- Gravity is the weakest fundamental interaction.

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GRAVITY = GEOMETRY

$$G_{\mu\nu} = T_{\mu\nu}$$

Relativity at a Young Age

Special Relativity

Postulates

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- Speed of light is constant

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- Physics is the same for all inertial observers

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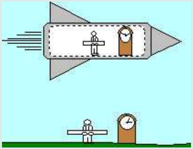
Consequences



Special Relativity

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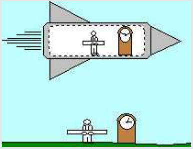
- Simultaneity



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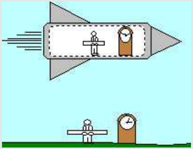

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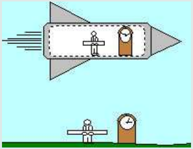

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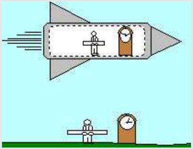
- $E = mc^2$ and more!



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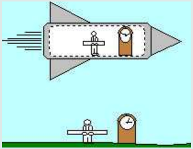

Noted Concerns

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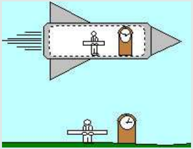

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- Reference Frames vs Coordinate Systems

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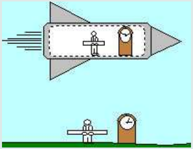

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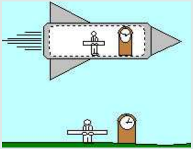
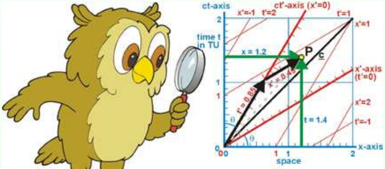


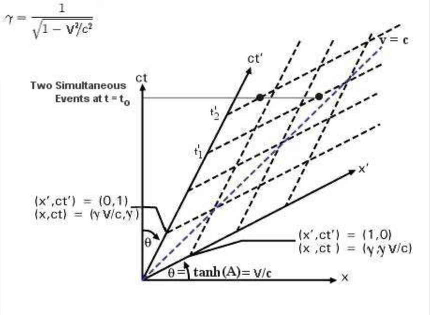
Noted Concerns

- Reference Frames vs Coordinate Systems
- Simultaneity Misconceptions
- No Spacetime Diagrams

Minkowski Spacetime



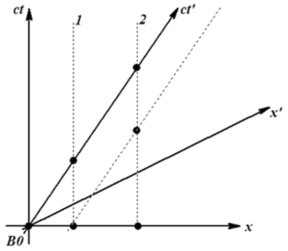
A two dimensional x,t Minkowski space-time diagram for a relative speed of v/c = 0.6c. The angle θ = arctan v/c. Here a point P in space-time, is located using rectangular coordinates, (green arrows), and the Lorentz coordinates, (black arrows).

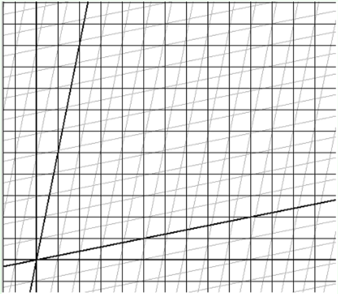


Spacetime Diagram Application

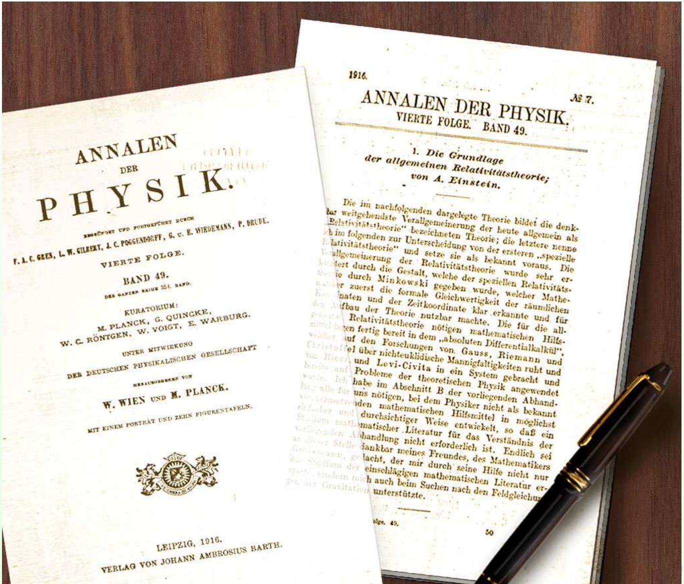
Train in Tunnel Problem

A relativistic train of rest length 240 meters travels at 0.6c through a tunnel which has rest length 360 meters. In the figure below the world lines for the tunnel openings are drawn as line 1 and 2 and the world line of the front of the train is the third dotted line. Let S_{tunnel} be the tunnel with coordinates (x, t) and let S_{train} be the train coordinates (x', t'). We set the origin as the event B0, the back of the train location just as the front end enters opening 1.





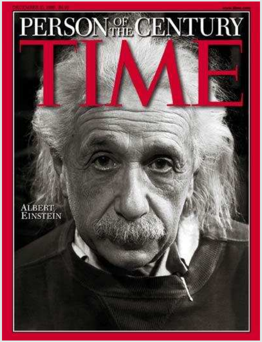
General Relativity - 1916



The Problem of the Century

from the Person of the Century - Smarr, 2000

- 1910s General Theory; Schwarzschild
- 1920s Equation of Motion Posed
- 1930s Two Body Problem Posed
- 1940s Cauchy Problem Posed
- 1950s Numerical Relativity Conceived
- 1960s Geometrodynamics; First NR Attempts
- 1970s Head-On Spacetime Roughed Out
- 1980s NR Becomes a Field
- 1990s Head-On Nailed; 3D Dynamics Begins



- 2000s 3D Dynamics Nailed; Gravitational Wave Astronomy

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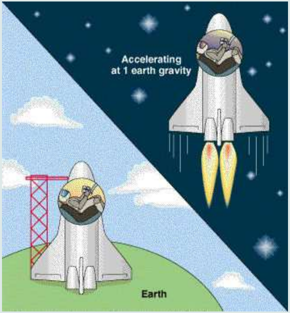
- **Spacetime Curvature Caused by Stress-Energy** - Described by Einstein field equations.

General Relativity

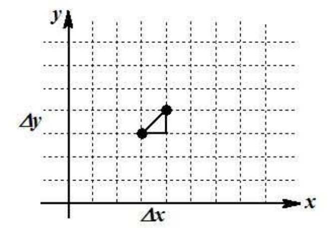
The Equivalence Principle

Experiments in (sufficiently small) freely falling laboratory, over a short time, give results that are indistinguishable from those experiments in an inertial frame in empty space



Spacetime

Flat Spaces - Cartesian



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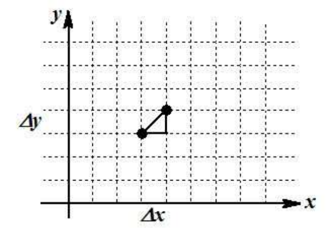
  $dS^2 = dx^2 + dy^2$



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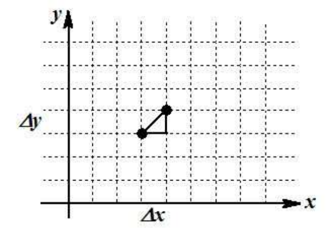
  $L = \sqrt{\left(\frac{dx}{d\sigma}\right)^2 + \left(\frac{dy}{d\sigma}\right)^2}$



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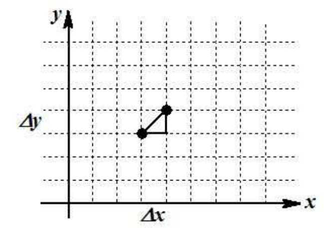

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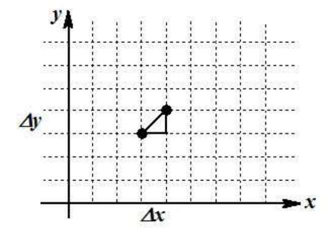

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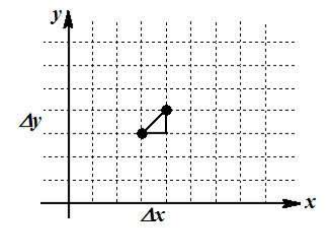
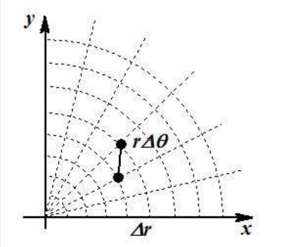
- **Geodesics**
  $\frac{d^2x}{dS^2} = 0, \quad \frac{d^2y}{dS^2} = 0$



Spacetime

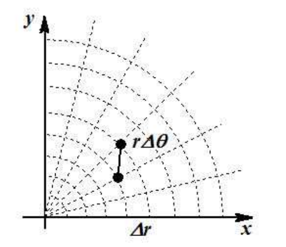
Flat Spaces - Polar



Spacetime

Flat Spaces - Polar

- **Line element**
  $dS^2 = dr^2 + (rd\theta)^2$



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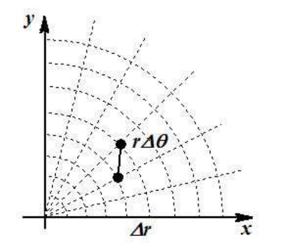

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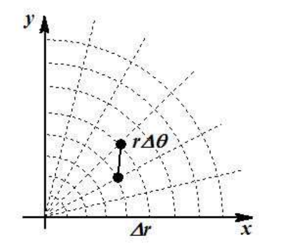

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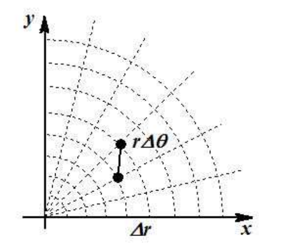
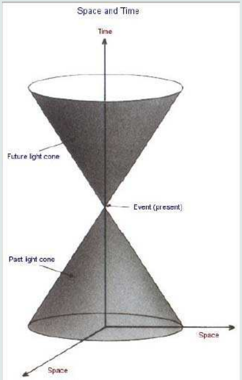
- **Geodesics**
  $\frac{\partial^2 r}{\partial S^2} = r\left(\frac{d\theta}{dS}\right)^2$,
  $\frac{d}{dS}\left(r^2 \frac{d\theta}{dS}\right) = 0.$



Spacetime

Minkowski Space



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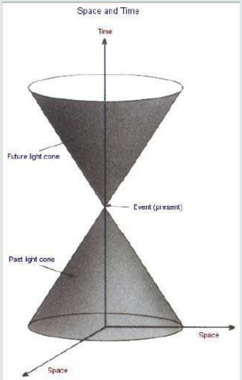

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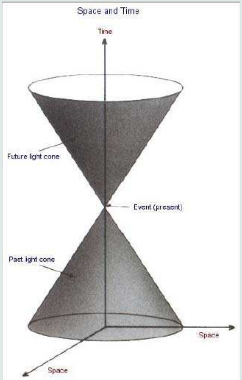

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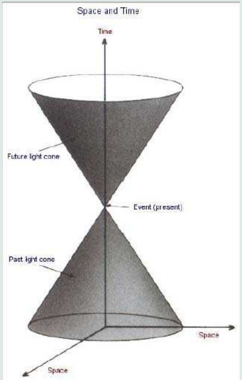

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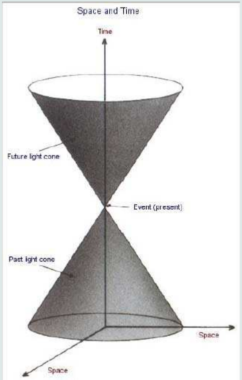

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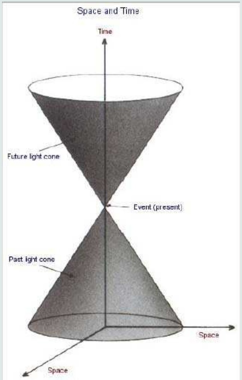

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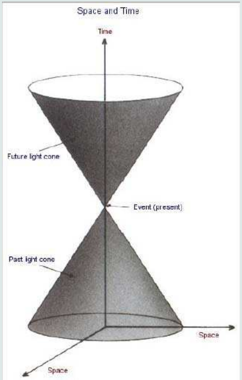
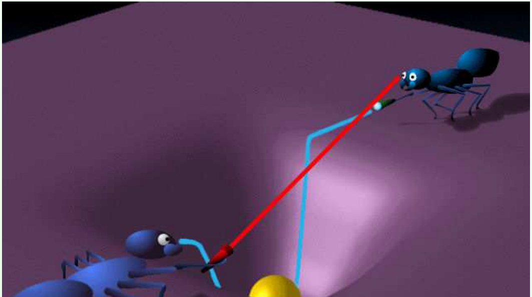


Spacetime

Curved Spaces



Spacetime

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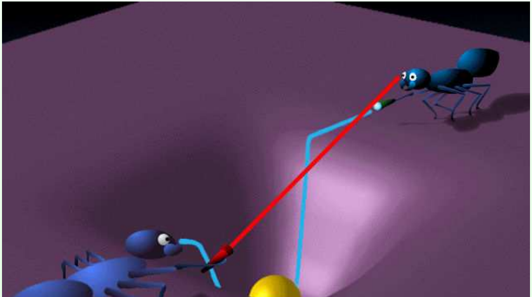
- **Line element** $ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$



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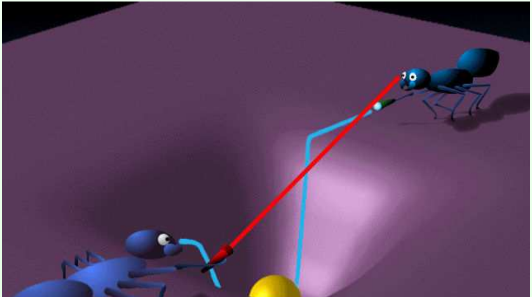

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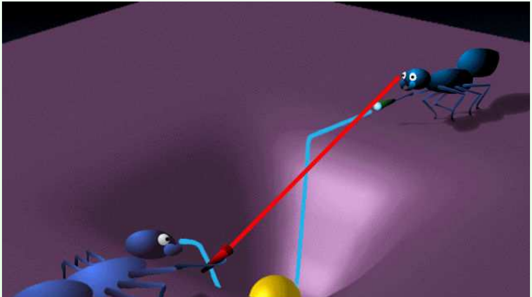

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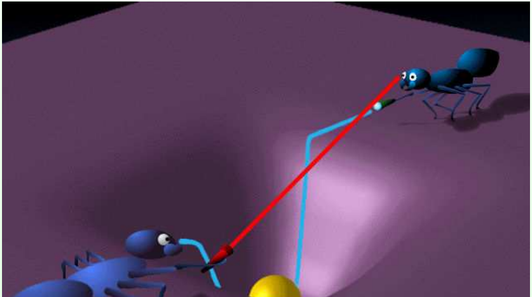

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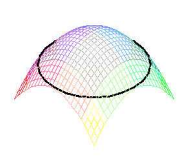
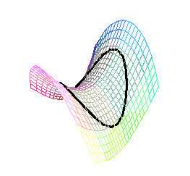
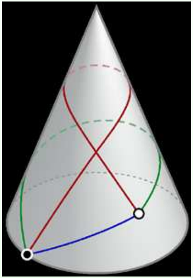
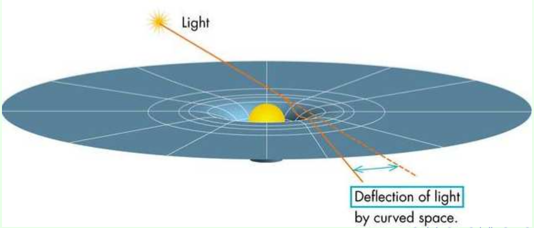






Spacetime

Schwarzschild



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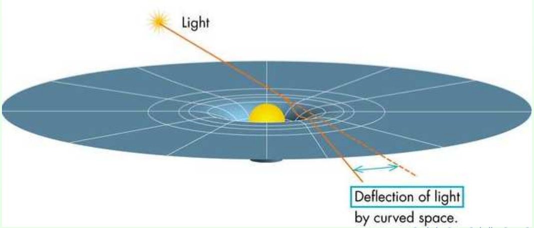
- **Line element** $ds^2 = -\left(1 - \frac{2M}{a}\right) dt^2 + \left(1 - \frac{2M}{a}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$



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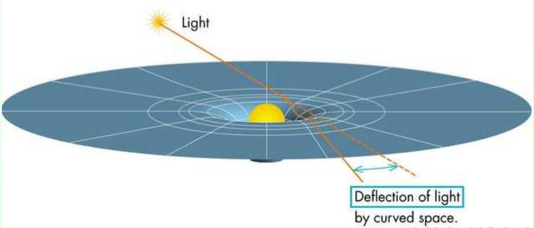

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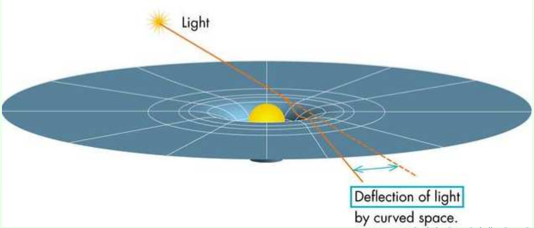

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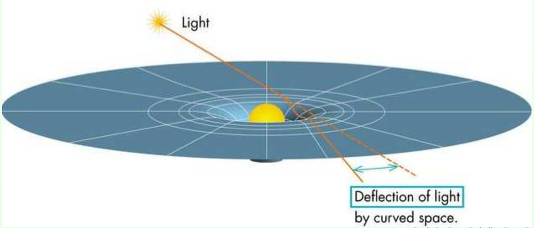
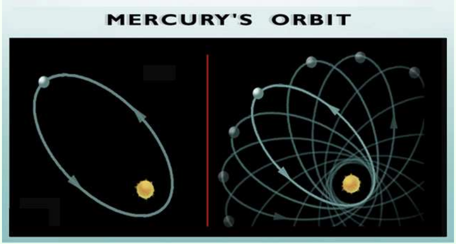
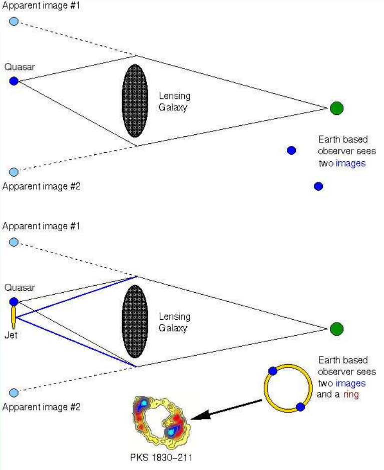
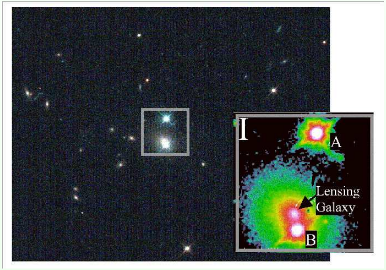


Classical Tests - Perihelion Shift

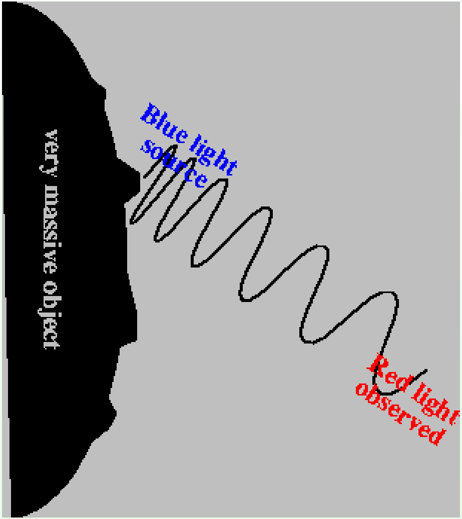


Classical Tests - Bending of Light

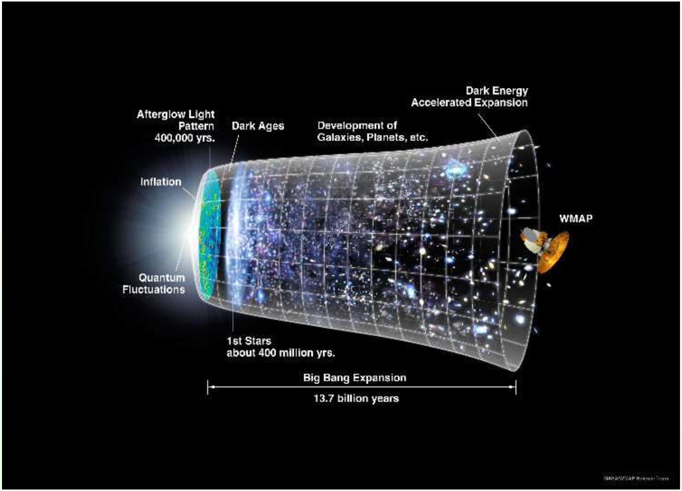




Classical Tests - Gravitational Red Shift

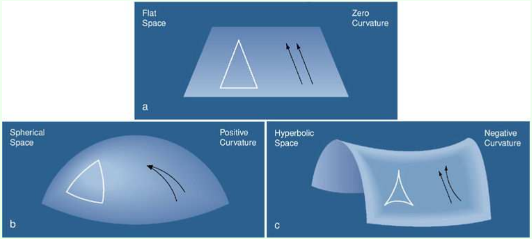


The Big Bang!


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
400,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

Cosmology

FRW model



Cosmology

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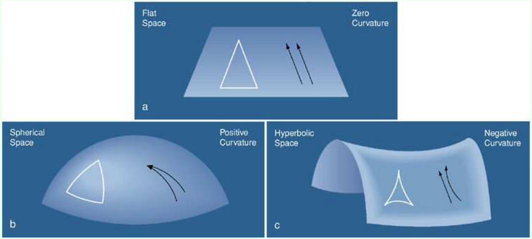

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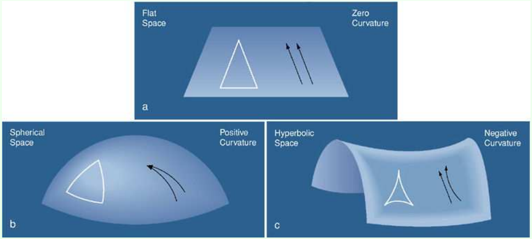

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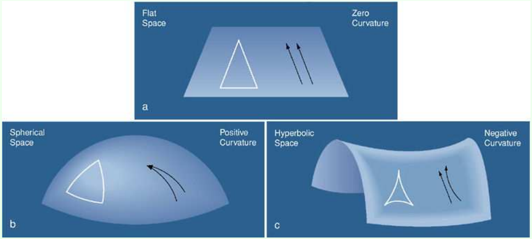

Applications of General Relativity

There are many more applications ...

Global Positioning System (GPS)	Lense-Thirring precession of a gyroscope
Cosmological redshift	Bending of light by the Sun
Big-bang	Shapiro time delay
Gravitational lensing	The fate of the universe
Propagation of gravitational waves	Determining parameters of binary pulsars
Spherical gravitational collapse	X-ray sources
Active Galactic Nuclei	Hawking radiation from black holes
Frame-dragging by a rotating body	Expansion of the universe
Gravitational redshift	Accretion disks around compact objects
Formation of black holes	Operation of gravitational wave detectors
Neutron stars	Precession of Mercury's perihelion
Cosmic Background Radiation	. . .

Syllabi - General Interest

The Narrative

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Syllabi - General Interest

Syllabus

Topics	Input	GR Application
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Light Shifting	Doppler	GPS with corrections. Doppler shifts, Reference Frames
Light bending	Why doesn't ISS fall? Orbits	Correct bending, Lensing
Black Holes	Spectral Lines, Escape Velocity	Maser data
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Syllabi - Physics First

The Logical Order

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- More flexible timing

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- More flexible timing
- More closely connected to curriculum - Mechanics!

Connections to Mechanics

First Integrals

Connections to Mechanics

First Integrals

- Test particles in Schwarzshild metric.

$$E = \frac{1}{2}\left(\frac{dr}{d\tau}\right)^2 + V_{\text{eff}}, \quad V_{\text{eff}} = -\frac{M}{r} + \frac{\ell^2}{2r^2} - \frac{M\ell^2}{r^3}$$

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- FRW Cosmological Models.
  $$\frac{1}{2}\left(\frac{da}{dt}\right)^2 + U_{\text{eff}} = \frac{\Omega_c}{2}, \quad U_{\text{eff}} = -\frac{1}{2}\left(\Omega_v a^2 + \frac{\Omega_m}{a} + \frac{\Omega_r}{a^2}\right)$$

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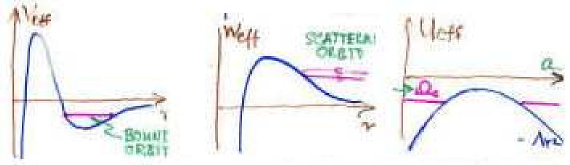
  $E = \frac{1}{2}\left(\frac{dr}{d\tau}\right)^2 + V_{\text{eff}}, \quad V_{\text{eff}} = -\frac{M}{r} + \frac{\ell^2}{2r^2} - \frac{M\ell^2}{r^3}$

- Light rays in Schwarzshild metric.

  $\frac{1}{b^2} = \frac{1}{\ell^2}\left(\frac{dr}{d\lambda}\right)^2 + W_{\text{eff}}, \quad W_{\text{eff}} = \frac{1}{r^2}\left(1 - \frac{2M}{r}\right)$

- FRW Cosmological Models.

  $\frac{1}{2}\left(\frac{da}{dt}\right)^2 + U_{\text{eff}} = \frac{\Omega_c}{2}, \quad U_{\text{eff}} = -\frac{1}{2}\left(\Omega_v a^2 + \frac{\Omega_m}{a} + \frac{\Omega_r}{a^2}\right)$



Syllabi - Physics First

Things to be included

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Course Themes

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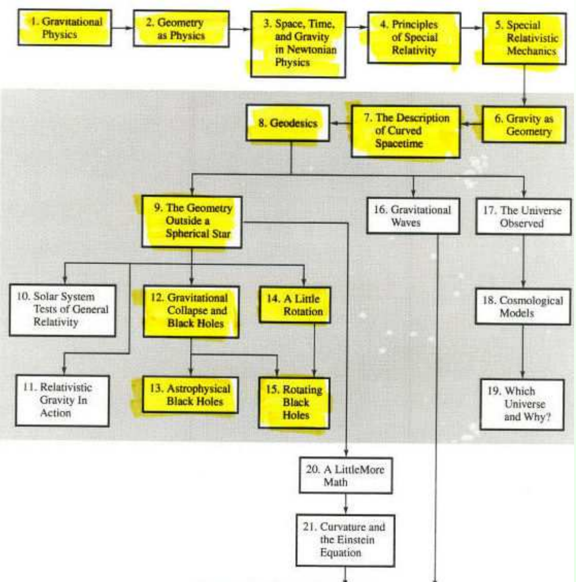
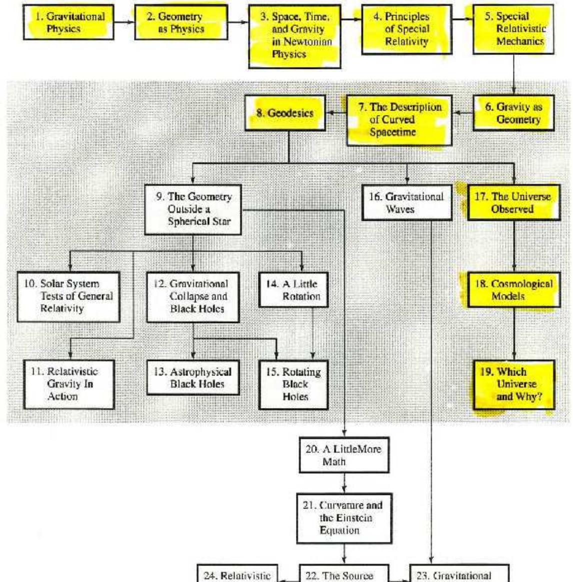
Course Themes

- Black Holes
- Cosmology
- Gravitational Waves

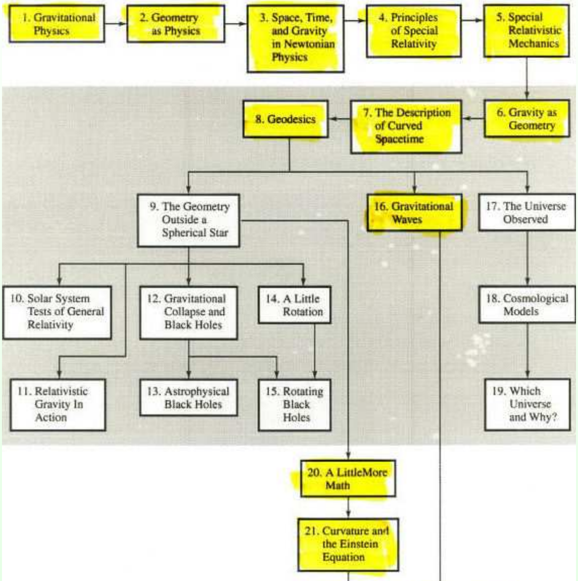
Syllabus - Black Hole Emphasis



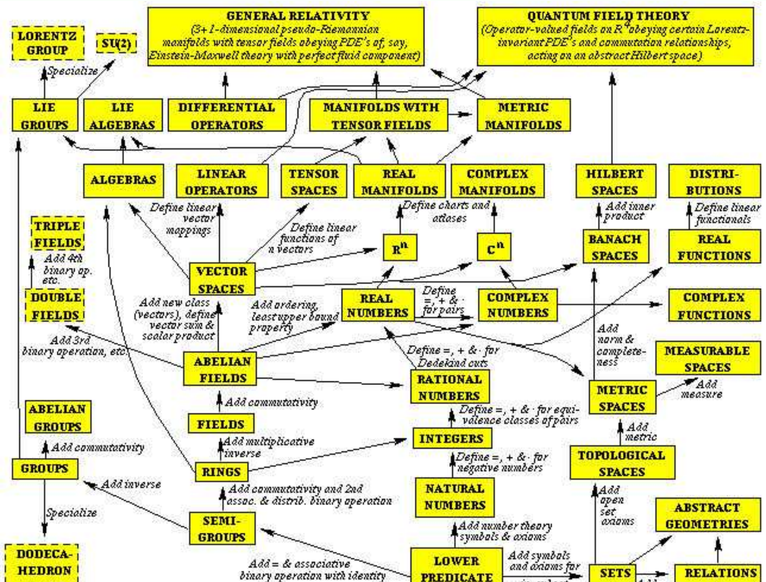
Syllabus - Cosmology Emphasis



Syllabus - Gravitational Wave Emphasis



Syllabi - Math Intensive



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The 7-Fold Way - Tom Moore

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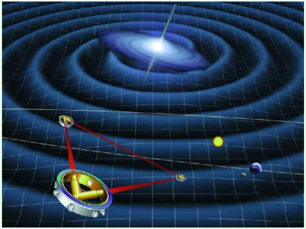
Treat Tensors as Generalized Vectors

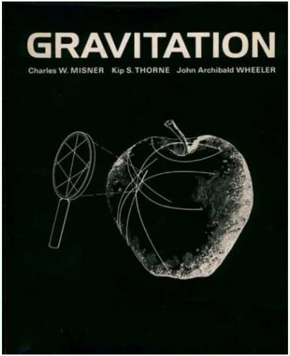
- Tensors represent physical objects
- Tensors have components relative to a basis
- Raise/lower indices to embed metric
- Tensors provide firm foundation for understanding

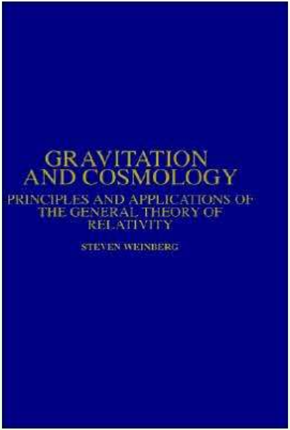
Syllabi - Math Intensive

1	Conceptual Overview	Review of Relativity	Four-Vectors
2	**Index Notation**	**Arbitrary Coordinates**	**Tensor Equations**
3	Maxwell's Equations	**Geodesics**	The Schwarzchild Metric
4	Particle Orbits	Perihelion Precession	Photon Orbits
5	Gravitational Lenses	Event Horizon	Alternative Coordinates
6	BH Thermodynamics	The Kerr Metric	Kerr Particle Orbits
7	Ergoregion and Horizon	Negative Energy Orbits	The Penrose Process
8	**The Absolute Gradient**	**Geodesic Deviation**	**The Riemann Tensor**
9	**Stress Energy Tensor**	**The Einstein Equation**	Interpreting the Equation
10	Schwarzchild Solution	The Observed Universe	A Cosmic Metric
11	Evolution of the Universe	Cosmic Implications	The Early Universe
12	**Linearized Gravity**	**Gauge Freedom**	Gravitational Waves
13	"Energy" in GWs	Generation of GWs	Applications

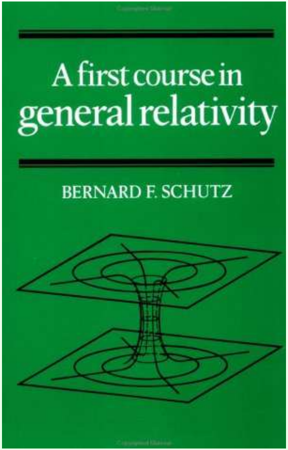


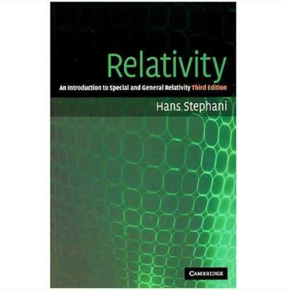
Texts - Old


GRAVITATION
Charles W. MISNER Kip S. THORNE John Archibald WHEELER

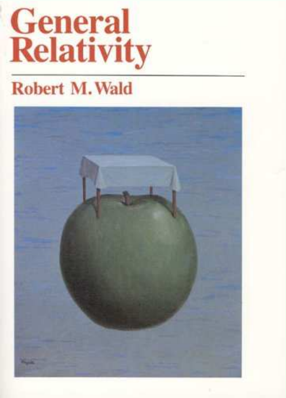

GRAVITATION AND COSMOLOGY
PRINCIPLES AND APPLICATIONS OF THE GENERAL THEORY OF RELATIVITY
STEVEN WEINBERG

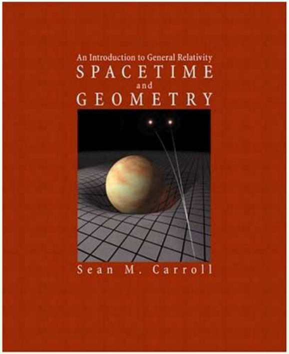
Texts - Semi-Old



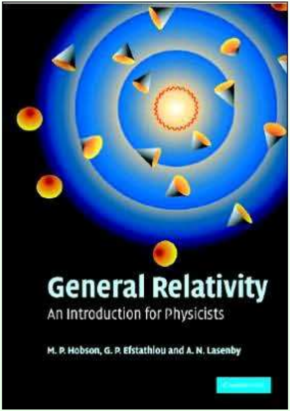


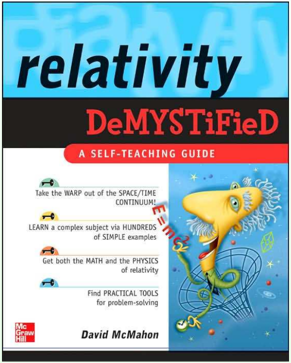
Texts - Math Intensive



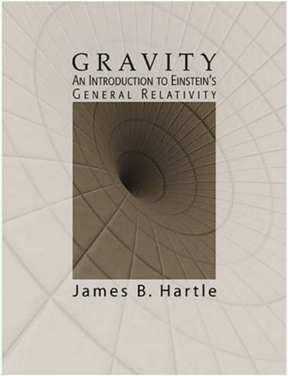


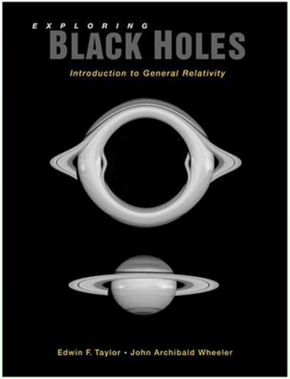
Texts - Math Intensive



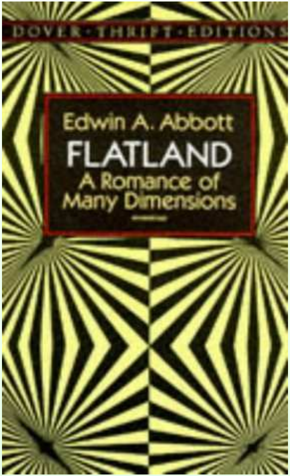


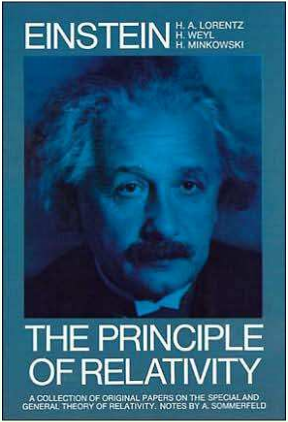
Texts - Physics First





Texts - General Interest





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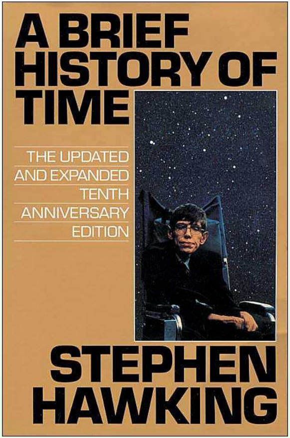
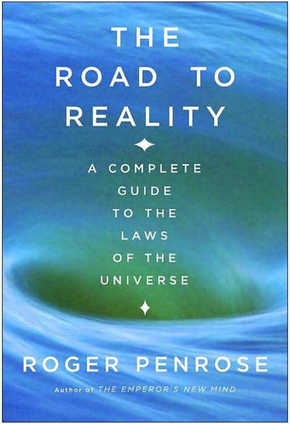
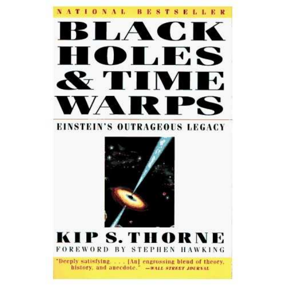
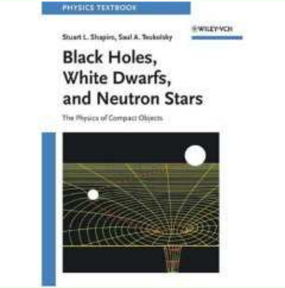
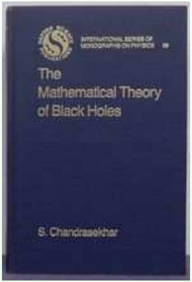




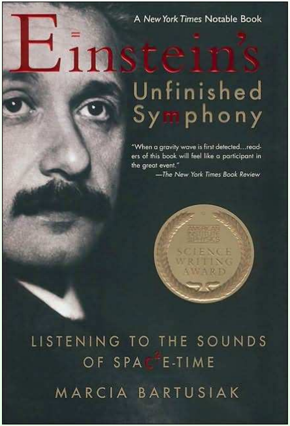
Texts - Black Holes

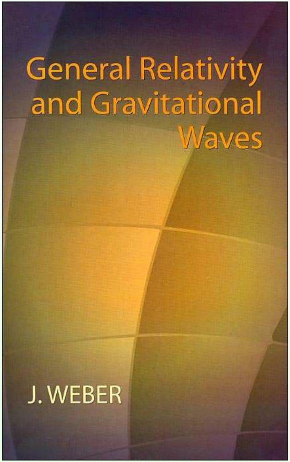






Texts - Gravitational Waves





Poster Presentation

Lessons on Teaching Undergraduate General Relativity and Differential Geometry Courses

Russell L. Herman and Gabriel Lugo

University of North Carolina Wilmington, Wilmington, NC

Abstract

We describe the course content and lessons learned teaching simultaneously offered courses to undergraduate physics and mathematics majors. A subset of students took both courses. The general relativity course was offered in the physics curriculum and focused more on the physics with standard mathematics prerequisites. The differential geometry course was aimed at the geometry of curves and surfaces ending with a study Cartan's equations and applications to computing curvatures in general relativity.

GR Syllabus

General Relativity

Geometry on a Sphere	Curved Spacetime
Special Relativity	Geodesic Equation
Four Vectors	Symmetries and Conservation Laws
Dynamics	Schwarschild Solution
Principle of Equivalence	Gravitational Redshift
Newtonian Gravity	Perihelion Shift
Metrics	Black Holes
Light Cones	Cosmology
Local Inertial Frames	Einsteins Equation

Prerequisites: Multivariate Calculus, Classical Mechanics, Modern Physics, Jr-Sr Standing.

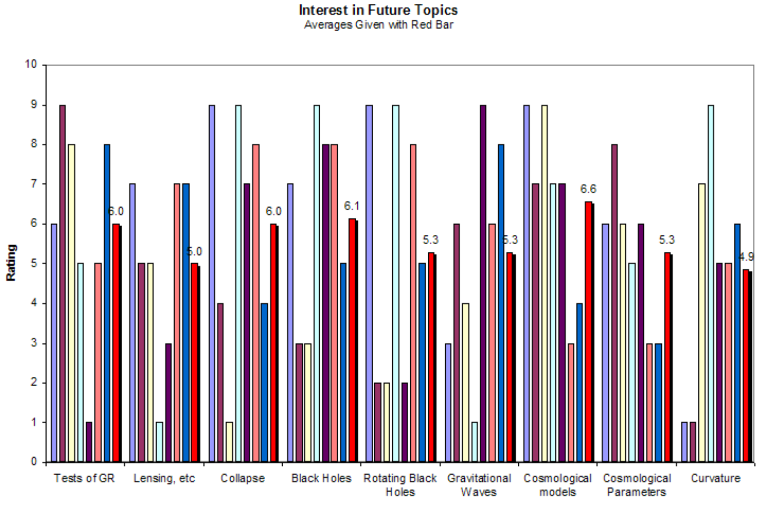
DG Syllabus

Differential Geometry

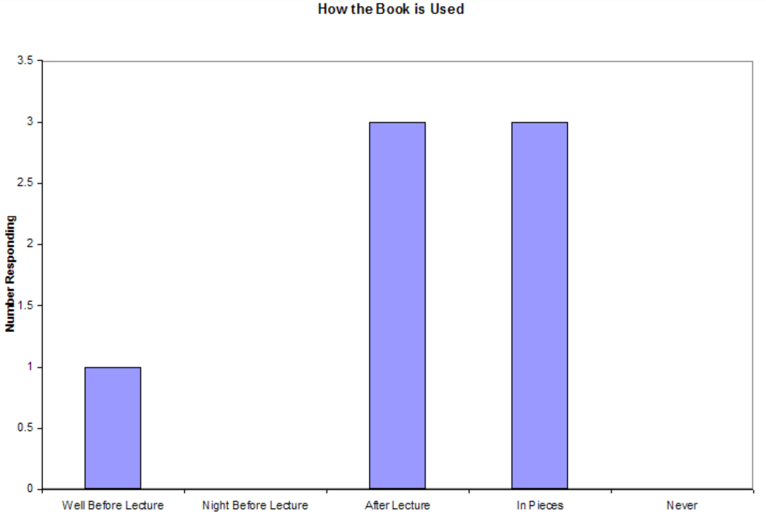
Linear Algebra	Exterior Derivatives
Tangent Vectors	Hodge * Operator
Curves	Frames
Fundamental Theorem of Curves	Curvilinear Coordinates
Surfaces	Covariant Derivatives
Curvature of Curves and Surfaces	Cartan Equations
1-Forms	Manifolds
Tensors	Fundamental Forms
Higher Rank Forms	Curvature and Einsteins Equation

Prerequisites: Linear Algebra, Multivariate Calculus, Jr-Sr Standing.

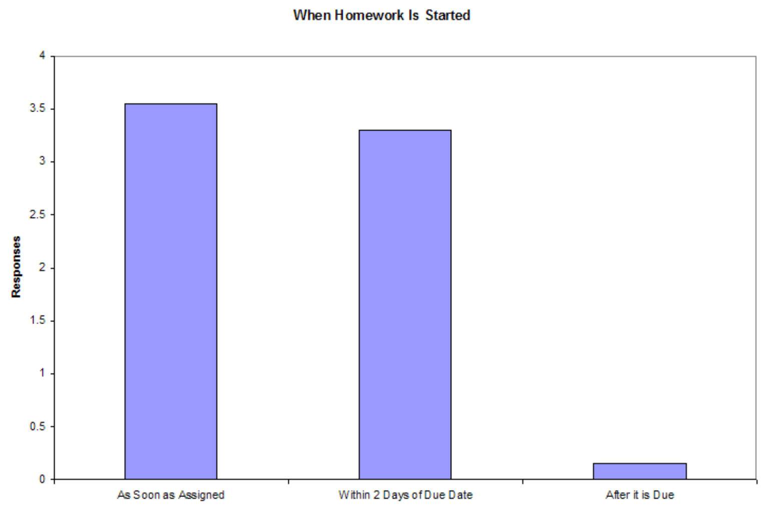
Midterm Survey - Topics Interest



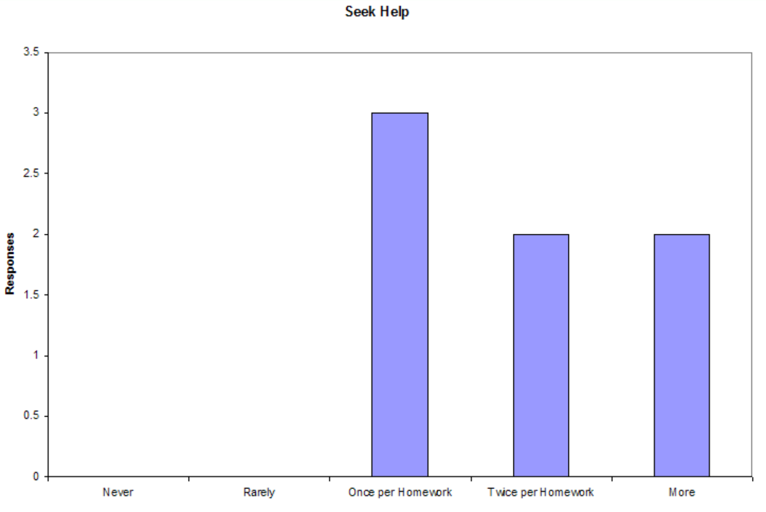
Midterm Survey - When Do Students Read Text?



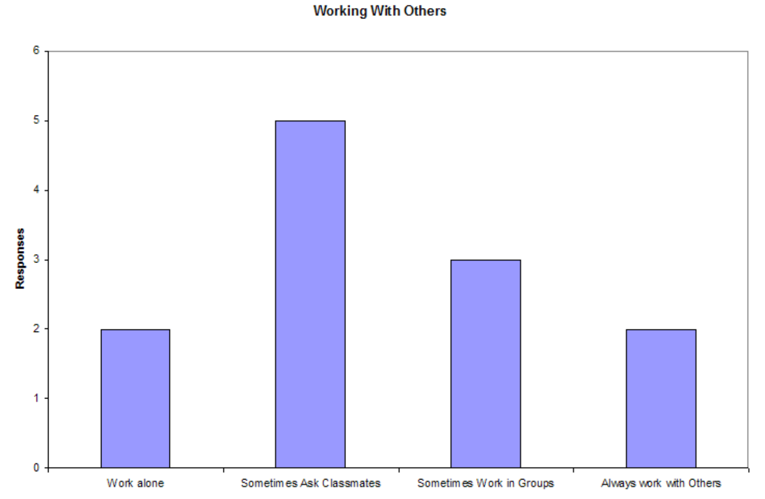
Midterm Survey - When Do They Start Assignments?



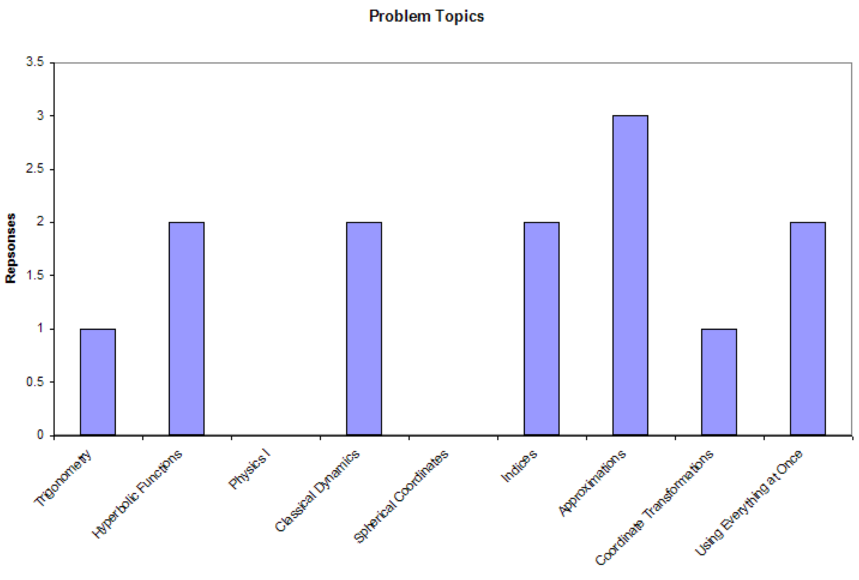
Midterm Survey - Do They Seek Help?



Midterm Survey - Do They Work Together?



Midterm Survey - What Gives Them Difficulty?



Lessons Learned

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Undergraduates need

1. more linear algebra emphasizing linear transformations, the spectral theorem and applications
2. more exposure to using approximations based on binomial expansions
3. more geometric insight
4. more exposure to indexed quantities
5. more practice doing homework in physics classes
6. lessons on how to read physics and mathematics texts
7. to learn how to transfer knowledge between courses

Decision Questions

You and Your Department

- How much time is available? 15 wks? 30 wks
- Is this one shot or ongoing?
- What is your background in GR?
- What subtopic is of most interest to you?

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The Students

- What is their background? Major/Non-major? Mechanics? E& M? Astrophysics? Beginning string theory?
- What is Math Background? Advanced Calculus? Differential Geometry?
- What are their motivations? - General interest, astrophysics, gravitational waves, intro to strings, just to know tensors?

Decision Questions

The Content

- Is the course to be a prerequisite?
- What do students need/want to hear? Solving Einstein equation or doing specific application?
- What is the purpose? What do you know well?
- Is there a focus? GWs, BHs, Cosmology, Formalism?

Summary

1. TGRU Workshop
   - Web Site
   - Why? Who? What?
2. Special Relativity
   - Postulates
   - Spacetime Diagrams
3. General Relativity
   - Principles
   - Flat Space
   - Curved Space
4. Course Syllabi
   - General Interest
   - Physics First
   - Math Intensive
5. Textbooks
6. Poster
7. Summary